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Original article

SPDEs driven by additive and multiplicative white noises: A numerical study

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Abstract

In this paper we present a finite element approximation of a model of a linear stochastic partial differential equation driven by additive and multiplicative white noises. Using the Wick-product properties and the Wiener–Itô chaos expansion, the stochastic variational problem is reformulated as a set of deterministic variational problems. To obtain the chaos coefficients in the corresponding deterministic equations, we use the usual Galerkin finite element method. Once this approximation is performed, the statistics of the numerical solution can be easily evaluated.

Keywords: Pressure equation; Wick calculus; Finite element method; Galerkin method; Stochastic simulation

1. Introduction

The purpose of this paper is to give a finite element approximation of a linear stochastic partial differential equation (SPDE) in infinite dimension driven by a white noise under the framework of the white noise analysis. We study the following problem: Find a solution $u(t, x, \omega)$ of the linear SPDE

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nabla\cdot(\kappa\lozenge\nabla u) - ru\lozenge W_1(t,\omega) = f + \sigma W_2(t,x,\omega) & \text{in } (0,T)\times\mathcal{D}\times\Omega \\ u(t,x,\omega) = 0 & \text{on } [0,T]\times\partial\mathcal{D}\times\Omega \\ u(0,x,\omega) = g(x,\omega) & \text{in } \mathcal{D}\times\Omega \end{cases} \tag{1}$$

where u is a scalar field, f is the source term, g is the initial condition, κ is the diffusion of the medium, W_1, W_2 are singular white noises, Ω is the set of random events, $\mathcal{D}\subset\mathbb{R}^d$ (d = 1, 2 or 3) an open bounded domain with a smooth boundary $\partial\mathcal{D}$ and $r, \sigma\in\mathbb{R}_+$ which represent the intensities of the reaction and the the white noise terms respectively. The term $ru \lozenge W(t, \omega)$ can be viewed as a stochastic perturbation of the reaction term ru.

Eq. (1), is a linearized model for the evolution of a scalar field in a space–time dependent random medium. It arises in several physical and mathematical problems such as:

- Flow in a porous medium where $u(t, x, \omega)$ denotes the pressure, κ is the permeability of the medium, f represents the external forces (for example sources or sinks in an oil-reservoir) and g represents the initial data. We allow κ,

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