

Original article

# On the global existence for a degenerate elliptic–parabolic seawater intrusion problem<sup>☆</sup>

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## Abstract

We studied a sharp interface model issuing from a seawater intrusion problem in a confined aquifer. This model consists in a coupled system of an elliptic and a de-generate parabolic equation. The global in time existence is obtained in using Schauder theorem combined with parabolic regularization. We establish a priori estimations for the P1 finite element method applied to this problem. Finally, we propose numerical simulations in easy examples.

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## 1. Introduction

The motivation of this paper is to get an efficient and accurate model to simulate the movement of the saltwater front in coastal aquifers.

Since freshwater and saltwater are miscible fluids, we have a transition zone separating them caused by hydrodynamic dispersion.

In a first approach, we will assume that saltwater and freshwater are no miscible and the domains occupied by each fluid are separated by an interface called sharp interface.

This modelling approach does not describe the nature and the behavior of the transition zone but does give information concerning the movement of the saltwater front. This information is important for the control of seawater intrusion and for the optimal exploitation of fresh groundwater.

Then the evolutions of the depth of the interface and of the freshwater hydraulic head are given by a coupled two-dimensional system consisting of an elliptic and a degenerate parabolic equations. We refer to Ref. [3] for more details about sea intrusion problem with sharp interface approach.

Let us mention that this seawater intrusion problem has been treated in Ref. [2]. The authors prove an existence result using the technique developed by Alt and Luckaus [1] combined with a time discretization. In this paper, we propose a

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