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A hole filling method for surfaces by using $\mathcal{C}^1$-Powell–Sabin splines. Estimation of the smoothing parameters

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Abstract

Let $D \subset \mathbb{R}^2$ be a polygonal domain, H be a subdomain of D and $f \in \mathcal{C}^1(\bar{D} - \overset{\circ}{H})$. In this paper we propose a method to reconstruct f over the whole $\bar{D}$ using a technique based on the minimization of an energy functional $\mathcal{J}$. More precisely, we construct a new $\mathcal{C}^1$-Powell–Sabin spline function f^* over $\bar{D}$ that approximates f outside H, and fills the hole of f inside H. The resulting filling patch strongly depends on the values of two smoothing parameters involved in the functional $\mathcal{J}$. We give a criteria to select optimum values of the parameters and we present some graphical and numerical examples.

Keywords: Filling holes; Minimal energy; Optimum parameters; Powell–Sabin

1. Introduction

In the last few years, several variational methods have been developed in relation with the approximation or interpolation a given set of scattered data. Most of these variational approaches consist of minimizing an energy functional that usually contains two terms: the first indicates how well the curve or surface approximates or interpolates the data set, while the second controls the degree of smoothness or fairness of the curve or surface. A wide range of minimization functionals have been proposed, derived from physical considerations (e.g. stretch energy or bending energy) or geometric entities (e.g. curve length, surface area or curvature). Discrete smoothing D^m-splines [1,2] and discrete smoothing variational splines [12] provide specific examples of variational curves and surfaces. These splines minimize, in a finite element space, some quadratic functionals that contain terms associated with Sobolev seminorms. These variational methods have received considerable attention in the last few years, due to their efficiency and usefulness in the fitting and design of curves and surfaces.

On the other hand, the problem of filling holes or completing a 3D surface arises in all sorts of computational graphics areas, like CAGD, CAD–CAM, Earth Sciences, computer vision in robotics, image reconstruction from satellite and radar information, etc. Several works related to the field of filling holes have been published in the last few years, e.g., [5,8,10,16]. In [8], the authors show the existence of a unique $\mathcal{C}^1$-cubic spline over the hole that minimizes an energy functional and satisfies certain interpolating conditions on the vertices of the triangulation of the hole. The final

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