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Quasi-Monte Carlo methods for Markov chains with continuous multi-dimensional state space

R. El Haddad [a,*], C. Lécot [b], P. L'Ecuyer [c], N. Nassif [d]

[a] *Département de Mathématiques, Université Saint-Joseph, BP 11-514, Riad El Solh Beyrouth 1107 2050, Lebanon*
[b] *LAMA, UMR 5127 CNRS & Université de Savoie, 73376 Le Bourget du Lac, France*
[c] *Département d'Informatique et de Recherche Opérationnelle, Université de Montréal, CP 6128, Succ. Centre-Ville, Montréal, H3C 3J7, Canada*
[d] *Department of Mathematics, American University of Beirut, BP 11-0236, Riad El-Solh Beyrouth 1107 2020, Lebanon*

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Abstract

We describe a quasi-Monte Carlo method for the simulation of discrete time Markov chains with continuous multi-dimensional state space. The method simulates copies of the chain in parallel. At each step the copies are reordered according to their successive coordinates. We prove the convergence of the method when the number of copies increases. We illustrate the method with numerical examples where the simulation accuracy is improved by large factors compared with Monte Carlo simulation.

Keywords: Markov chain; Discrepancy; Quasi-Monte Carlo method; Simulation

1. Introduction

Many real-life systems can be modeled using Markov chains. Fields of application are queueing theory, telecommunications, option pricing, etc. In most interesting situations, analytic formulas are not available and the state space of the chain is so large that classical numerical methods would require a considerable computational time and huge memory capacity. So Monte Carlo (MC) simulation becomes the standard way of estimating performance measures for these systems. A drawback of MC methods is their slow convergence. One approach to improve the accuracy of the method is to change the random numbers used. Quasi-Monte Carlo (QMC) methods use quasi-random numbers instead of pseudo-random numbers. Pseudo-random numbers aim to simulate a sequence of independent and identically distributed (i.i.d.) random variables with a given distribution (we only consider the uniform distribution). In the example of MC integration, it is not so much the randomness of the samples that is relevant, but rather that the samples should be spread in a uniform manner over the integration domain. Quasi-random numbers are sample points for which the empirical distribution is close to the uniform distribution; unlike for random sampling, quasi-random points are not required to be independent and may be completely deterministic.

* Corresponding author. Tel.: +961 1 421 372; fax: +961 4 532 657.
E-mail addresses: rami.haddad@fs.usj.edu.lb (R. El Haddad), christian.lecot@univ-savoie.fr (C. Lécot), lecuyer@iro.umontreal.ca (P. L'Ecuyer), nn12@aub.edu.lb (N. Nassif).

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