Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 81 (2011) 2244–2257
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

Linear least squares parameter estimation of nonlinear reaction diffusion equations

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Received 16 November 2009; received in revised form 10 February 2011; accepted 9 May 2011
Available online 20 May 2011

Abstract

This paper concerns with the development of a direct parameter identification procedure for a class of nonlinear reaction–diffusion equations. We assume to know the model equations with the exception of a set of constant parameters, such as diffusivity and reaction term parameters. Using the finite element method the original partial differential equation is transformed into a set of ordinary differential equations. A linear least squares method is then applied to estimate the unknown parameters by using normal equations. The measurements errors obtained following this approach are significantly lower than the error obtained by a nonlinear least squares identification procedure. In order to better understand the differences between the two approaches, a sensitivity analysis with respect to initial conditions and mesh dimension is performed. The robustness of the method is tested on noise corrupted data, showing that the linear least square method may be sensitive to perturbations in the data. The procedure is applied to two ecological models describing the dynamics of population growth.

Keywords: Nonlinear reaction–diffusion equations; Parameter estimation; Finite element discretization; Linear and nonlinear LS

1. Introduction

Reaction–diffusion equations describe many natural, physical, ecological, biochemical and social phenomena. Mathematical modelling of such systems often involves infinite dimensional dynamical systems described by partial differential equations (PDEs).

In practical applications, an important problem is the estimation of the unknown parameters of the equations in order to reproduce accurately the dynamics of the real processes. This task consists with finding the solution of suitable inverse problems involving the discretization of the system under investigation. The discretized version of reaction diffusion equations may consist with finite difference (see [8] for an application of this method to an ecological system) or finite elements models (see [10] for the finite element method), where, in the first case, a discretization of the equations is considered and, in the second case, a set of basis functions are selected for approximating the model solutions. In both numerical approximations, the parameter estimation of parabolic equations with linear and nonlinear reaction terms may involve nonlinear algorithms making difficult the application of system identification procedures.

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0378-4754/$36.00
doi:10.1016/j.matcom.2011.05.006