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ScienceDirect
Mathematics and Computers in Simulation 81 (2011) 1697–1706
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

The domain of attraction for the endemic equilibrium of an SIRS epidemic model

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Received 3 July 2009; received in revised form 7 August 2010; accepted 31 August 2010
Available online 16 December 2010

Abstract

In this paper, a new method is adopted to construct a Lyapunov function for the endemic equilibrium of the J. Mena-Lorca and H.W. Hothcote's SIRS epidemic model with bilinear incidence and constant recruitment. On the basis of the Lyapunov function, the domain of the attraction of the endemic equilibrium is estimated by solving an LMI optimization problem with multivariate polynomial objective function and constraints.

AMS Classification: 34D20; 94D30

Keywords: Epidemic model; Domain of attraction; LMI optimization; Lyapunov function; Stability

1. Introduction

Since long time ago, epidemic disease has endangered the world deeply (see [2]), and huge calamity has been brought to the humanity. As a result, millions of people die and social economics are heavily destroyed. Therefore, it is necessary to adopt all kinds of methods to control and eliminate epidemic disease.

Epidemic models can contribute to the design and analysis of epidemiological surveys, suggesting crucial data that should be collected, identifying trends, making general forecasts, and estimating the uncertainty in forecasts. Therefore, to construct epidemic models and then investigate its qualitative and quantitative properties has become a focus in applied mathematics. In fact, the original work on epidemic models is due to a Kermack and McKendrick's paper [14], where they assume the size of the total population is invariable, divide the population into three classes: the infective, the susceptible and the removed, and formulate an SIR epidemic model. Since then there have appeared various epidemic models (see [1–3,6,7,9,10,19,18,21] and references therein for examples) to describe the spread process of different diseases, lots of numerical and analytical results are obtained, which benefit the disease's controlling and defending.

It is well known that the epidemicity of disease is closely related to the stability of the solutions of mathematical models. Generally, the Lyapunov's second method is used to analyze the stability of epidemic model. But, sometimes

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0378-4754/$36.00 © 2010 Published by Elsevier B.V. on behalf of IMACS.
doi:10.1016/j.matcom.2010.08.012