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On the use of matrix functions for fractional partial differential equations

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Abstract

The main focus of this paper is the solution of some partial differential equations of fractional order. Promising methods based on matrix functions are taken in consideration. The features of different approaches are discussed and compared with results provided by classical convolution quadrature rules. By means of numerical experiments accuracy and performance are examined.

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1. Introduction

The development of models based on differential equations of non-integer order has recently gained popularity in the investigation of dynamical systems exhibiting an anomalous decay of nonexponential type. Problems of this type arise in many areas, ranging from physics to electrochemistry, biology, economics and probability theory (see [27] and references therein).

As a consequence, the solution of differential equations of fractional order (FDEs) represents nowadays an active research area for numerical analysts.

In this paper we consider linear systems of FDEs, with constant coefficients, in the form

$$D_{t_0}^{\alpha} U(t) = AU(t) + b, \qquad U^{(l)}(t_0) = U_0^l, \quad l = 0, \ldots, m, \tag{1}$$

where $\alpha > 0$ is the non-integer order, $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $U(t) : [t_0, T] \to \mathbb{R}^n$ and $m = \lceil \alpha \rceil$. Here $D_{t_0}^{\alpha}$ denotes the fractional derivative operator, with respect to the origin t_0, according to the Caputo's definition [27]

$$D_{t_0}^{\alpha} y(t) \equiv \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{y^{(m)}(u)}{(t-u)^{\alpha+1-m}}\, du,$$

with $\Gamma(\cdot)$ denoting the Euler's gamma function. It is a well-known result [27] that, for $0 < \alpha < 1$, the true solution of this problem can be expressed as

$$U(t) = U_0 + (t-t_0)^{\alpha} E_{\alpha,\alpha+1}(A(t-t_0)^{\alpha})(AU_0 + b), \tag{2}$$

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