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# Seasonality and mixed vaccination strategy in an epidemic model with vertical transmission

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**Abstract**

First vaccination (vaccinate at birth) and pulse vaccination are two methods used to control the spread of diseases as well as the elimination of them. Owing to the seasonal fluctuations in transmission of many diseases, we propose an impulsive SIRS epidemic model with periodic saturation incidence and vertical transmission. The effects of periodic varying contact rate and mixed vaccination strategy on eradication of infectious diseases are studied. A threshold for a disease to be extinct or endemic is established. Our results imply that the diseases will die out eventually if the basic reproduction number is less than unity, whereas the diseases will persist if the basic reproduction number is larger than unity. Finally, numerical simulations support our analytical conclusions.


*Keywords:* Seasonal variability; Mixed vaccination strategy; Epidemiology; Threshold

## 1. Introduction

Every year, infectious diseases affect millions of lives. So controlling the spread of the diseases as well as the elimination of them has been, and is still intensely and deeply studied by many scholars [19]. Vaccination as a approach of controlling or eradicating infectious diseases from the population is widely used, such as hepatitis B, yellow fever, rabies, polio, flu, measles, and tuberculosis. An immunization campaign carried out by the World Health Organization (WHO) from 1967 to 1977 eradicated the natural occurrence of smallpox [25]. The standard conventional approach has been constant vaccination, or a uniform and continuous effort of administering the vaccine to a population to try to stop the outbreak. Recently, another method, pulse vaccination, has been found to be more effective at eliminating epidemics. Pulse vaccination works by repeatedly applying a vaccine over defined age ranges [19], and at each vaccination time, a constant fraction of the susceptible population is vaccinated. Recently, many authors have studied the pulse vaccination effects on the controlling the spread of the diseases [8,10,17,18,6,20,21]. They try to find the threshold conditions that determine whether the diseases will outbreak to occur or die out. Many biomathematical and computational models have been introduced recently to study, for example, different virus dynamics [27], vaccination strategies [15], hepatitis A virus [1], hepatitis virus B [22,24] or cervical cancer

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