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Stability of the modified Craig–Sneyd scheme for two-dimensional convection–diffusion equations with mixed derivative term

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Abstract

The modified Craig–Sneyd (MCS) scheme is a promising splitting scheme of the ADI type for multi-dimensional pure diffusion equations having mixed spatial-derivative terms. In this paper we investigate the extension of the MCS scheme to two-dimensional convection–diffusion equations with a mixed derivative. Both necessary and sufficient conditions on the parameter θ of the scheme are derived concerning unconditional stability in the von Neumann sense.

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Keywords: Initial-boundary value problems; Convection–diffusion equations; Method-of-lines; ADI splitting schemes; Von Neumann stability analysis

1. Introduction

We consider the numerical solution of initial value problems for large systems of ordinary differential equations (ODEs),

$$U'(t) = F(t, U(t)) \quad (t \geq 0), \qquad U(0) = U_0, \tag{1.1}$$

with given vector-valued function F, given initial vector U_0, and unknown vectors $U(t)$ (for $t > 0$). Our interest in this paper lies in systems (1.1) that arise from semi-discretization of initial-boundary value problems for two-dimensional convection–diffusion equations possessing a mixed spatial-derivative term,

$$\frac{\partial u}{\partial t} = d_{11}u_{xx} + (d_{12} + d_{21})u_{xy} + d_{22}u_{yy} + c_1u_x + c_2u_y. \tag{1.2}$$

Here $c = (c_i)$ and $D = (d_{ij})$ denote a given real vector and a given positive semi-definite real matrix, respectively. A main application area of equations of the kind (1.2) is financial option pricing theory, where mixed derivative terms u_{xy} arise naturally since the underlying Brownian motions are usually correlated to each other. Extensive details and examples of financial applications are given in, for example, the references [11–13].

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