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Exponential Lawson integration for nearly Hamiltonian systems arising in optimal control[☆]

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Abstract

We are concerned with the discretization of optimal control problems when a Runge–Kutta scheme is selected for the related Hamiltonian system. It is known that Lagrangian's first order conditions on the discrete model, require a symplectic partitioned Runge–Kutta scheme for state–costate equations. In the present paper this result is extended to growth models, widely used in Economics studies, where the system is described by a current Hamiltonian. We prove that a correct numerical treatment of the state–current costate system needs Lawson exponential schemes for the costate approximation. In the numerical tests a shooting strategy is employed in order to verify the accuracy, up to the fourth order, of the innovative procedure we propose.

Keywords: Partitioned Runge–Kutta methods; Exponential Lawson schemes; Optimal growth models

1. Introduction

The scientific community is well used to the evidence that new techniques, independently introduced in different research fields, may sometimes be reduced to general frameworks, accounting for all the aspects that lead to their apparently different motivations and definitions. It is the case of an interesting work done by Bonnans and Varin [4], where they proved that the order conditions of Runge–Kutta schemes for finite horizon optimal control problems, determined by Hager [8], by hand, up to the fourth order, correspond to order conditions for a partitioned symplectic Runge–Kutta scheme. They approximate the solution of the optimal control problem by applying an s-stage Runge–Kutta solver and they prove that optimality necessary conditions related to the discrete model are satisfied by a symplectic partitioned scheme on the Hamiltonian system, where the control variables are explicitly expressed in terms of states and costates.

Symplectic integrators have been designed for the numerical solution of Hamiltonian systems with the aim of preserving the geometrical features of the theoretical dynamics (for details the reader is referred to [10]). A modern order accuracy theory for symplectic Runge–Kutta schemes can be found in Ref. [19] where a natural basis set of necessary and sufficient order conditions is provided. As concerns symplectic partitioned Runge–Kutta schemes, the order theory based on oriented free trees is given in Ref. [17], while the way of constructing the explicit expression of the order conditions is provided by Bonnans and Varin [4]. When dealing with explicitly time dependent Hamiltonian systems, the usual trick of adding the time as a new variable with dynamics $\dot{t} = 1$ allows to apply the same integrators as

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