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Locating coalescing singular values of large two-parameter matrices

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Abstract

Consider a matrix valued function $A(x) \in \mathbb{R}^{m\times n}$, $m \geq n$, smoothly depending on parameters $x \in \Omega \subset \mathbb{R}^2$, where Ω is simply connected and bounded. We consider a technique to locate parameter values where some of the q dominant ($q \leq n$) singular values of A coalesce, in the specific case when A is large and $m > n \gg q$.

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Notation. An $m \times n$ real matrix is indicated with $A \in \mathbb{R}^{m\times n}$. We always consider the 2-norm for vectors and matrices. A matrix valued function $A : \mathbb{R} \to \mathbb{R}^{m\times n}$, continuous with its first l derivatives ($l \geq 0$), is indicated as $A \in \mathcal{C}^l(\mathbb{R}, \mathbb{R}^{m\times n})$. If $l = 0$, we also simply write $A \in \mathcal{C}$. If $A \in \mathcal{C}^l(\mathbb{R}, \mathbb{R}^{m\times n})$ is periodic of (minimal) period $\tau > 0$, we write it as $A \in \mathcal{C}^l_\tau(\mathbb{R}, \mathbb{R}^{m\times n})$. With $\Omega \subset \mathbb{R}^2$ we indicate an open and bounded simply connected planar region, and $x = (x_1, x_2)$ will be coordinates in Ω. For a function $A(x)$, $x \in \Omega$, we will write $A \in \mathcal{C}^l(\Omega, \mathbb{R}^{m\times n})$ as appropriate.

1. Introduction

Consider a matrix $A \in \mathbb{R}^{m\times n}$, $m \geq n$. Recall that the – reduced – SVD (Singular Value Decomposition) of A is the decomposition

$$A = U\Sigma V^T,$$

where $U \in \mathbb{R}^{m\times n}$ is orthonormal ($U^TU = I_n$), $V \in \mathbb{R}^{n\times n}$ is orthogonal, and $\Sigma \in \mathbb{R}^{n\times n}$ is the diagonal matrix of the singular values: $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$. The SVD of a matrix is one of the most useful decompositions in linear algebra: the rank of A, as well as orthogonal representations for fundamental subspaces associated with A can be nicely retrieved from the factors of the SVD; see [5].

An important application of the SVD is that it is easy to obtain from it the best approximation (in the 2-norm) to the matrix A among matrices of a given rank. Namely, if $A = U\Sigma V^T$, and $\sigma_1 \geq \cdots \geq \sigma_q > \sigma_{q+1} \geq \cdots \geq \sigma_n$, then the best approximation of rank q to A is given by $U^{(q)}\Sigma^{(q)}(V^{(q)})^T$, where $U^{(q)} = U(:, 1:q)$, $V^{(q)} = V(:, 1:q)$ and

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