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Monte Carlo algorithms for evaluating Sobol' sensitivity indices

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Abstract

Sensitivity analysis is a powerful technique used to determine robustness, reliability and efficiency of a model. The main problem in this procedure is the evaluating total sensitivity indices that measure a parameter's main effect and all the interactions involving that parameter. From a mathematical point of view this problem is presented by a set of multidimensional integrals. In this work a simple adaptive Monte Carlo technique for evaluating Sobol' sensitivity indices is developed. A comparison of accuracy and complexity of plain Monte Carlo and adaptive Monte Carlo algorithms is presented. Numerical experiments for evaluating integrals of different dimensions are performed.

Keywords: Sensitivity analysis; Global sensitivity indices; Multidimensional numerical integration; Adaptive Monte Carlo algorithm

1. Motivation and discussion of some existing sensitivity analysis approaches

Mathematical modeling is the use of mathematics to describe real-world phenomena and has the following purposes—to investigate important questions about the observed world, to explain real-world phenomena, to test ideas, to make predictions about the real world. Mathematical models are used for simulation, when experiments are too expensive or even impracticable, and for prediction. They are utilized to approximate various highly complex engineering, physical, environmental, social, and economic systems. In some situations it is important to measure relations that describe the effect on an output when the conditions for the input change. These methods are collected under the name of **s** ensitivity **a** nalysis (SA). Applications for SA are all the processes where it is useful to know which variables mostly contribute to output variability. Two classes in sensitivity analysis have been distinguished: local SA and global SA. Local SA studies how some small variations of inputs around a given value change the value of the output. Global SA takes into account all the variation range of the inputs, and has for aim to apportion output uncertainty to inputs' ones. Global SA apportions the output uncertainty to the uncertainty in the input factors, covering their entire range space.

Our primary motivation for considering numerical algorithms for computing **s** ensitivity **i** ndices (SIs) is the study of environmental security using large-scale mathematical models like **Uni** fied **D** anish **E** ulerian **M** odel (UNI-DEM)

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