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On an approach to deal with Neumann boundary value problems defined on uncertain domains: Numerical experiments

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Abstract

Neumann boundary value problems for second order elliptic equations are considered on a 2D domain whose boundary is not known and might be even non-Lipschitz. Although the domain of definition is unknown, it is assumed that (a) it contains a known domain (subdomain), (b) it is contained in a known domain (superdomain), and (c) both the subdomain and superdomain have Lipschitz boundary. To cope with the Neumann boundary condition on the unknown boundary and to properly formulate the boundary value problem (BVP), the condition has to be reformulated. A reformulated BVP is used to estimate the difference between the BVP solution on the unknown domain and the BVP solution on the known subdomain or superdomain. To evaluate the estimate, the finite element method is applied. Numerical experiments are performed to check the estimate and its response to a shrinking region of uncertainty.

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Keywords: Uncertain boundary; Uncertain domain; Neumann boundary value problem; Estimate

1. Introduction

Since the uncertainty in the domain of definition of a boundary value problem (BVP) causes the uncertainty in the solution of that problem, methods are sought to describe at least some features of the uncertain solution.

Although BVPs on uncertain domains do not seem to be widely studied, some literature exists on this subject. However, stochastic sort of uncertainty is often assumed; see, e.g., [1,4,6,7]. As a consequence, stochastic methods are applied to infer stochastic characteristics of solutions.

In [2,3,5], the authors consider uncertainty in the domain caused by even more limited information. They assume that the available knowledge about an uncertain domain $\Omega \subset \mathbb{R}^2$ is as follows: (a) $\partial\Omega = \partial\bar{\Omega}$, i.e., the boundary of Ω coincide with the boundary of the closure of Ω; (b) $\bar{\Omega}_{\text{low}} \subset \Omega \subset \bar{\Omega} \subset \Omega_{\text{up}} \subset \bar{\Omega}_{\text{up}}$ where Ω_{low} and Ω_{up} are two known bounded domains with Lipschitz boundary. We will adhere to these assumptions, too.

Since Ω is, in fact, an unknown domain, the question arises how we can define the Neumann boundary condition (BC) on an unknown boundary. The answer is not obvious, but to get an insight, a natural approach is to analyze BVPs on sequences of domains and their limits.

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