

# The robustness of a Nash equilibrium simulation model: Game-theoretic approach using variable metric projection method

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## Abstract

This paper proposes a Nash equilibrium model that applies continuous time replicator dynamics to the analysis of oligopoly markets. The robustness of the proposed simple Nash equilibrium model under the simultaneous constraints of allocation of product and market share using a simulation method to derive an optimal solution for production decisions by rival firms in oligopoly markets is tested by changing profit and cost function parameters, as well as the initial production values and market shares of the firms examined in this study. The effects of differences in conjectural variation and initial allocation of market share on the convergent values are considered, particularly in the case of corner solutions. This approach facilitates the understanding of the robustness of attaining equilibrium in an oligopoly market.

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## 1. Introduction

Oligopoly markets prevail in developed countries in both the industrial and service sectors. See, for example, Dixit [3], which examines the U.S. automobile industry, and Klepper [4], which examines the airline industry. Competition and conjecture play an important role in improving profits in oligopoly firms. The decision-making process and solution under such circumstances has been studied in game theory. In the game-theoretic approach, one usually focuses on two problems: (1) What is a rational strategy regarding rational allocation in a market among agents that compete with each other in an oligopoly market? (2) How do they change their behavior in order to reach the rational allocation? The former is a static problem to obtain a desirable strategy under the conditions of competition and conjecture, while the latter is a dynamic problem to find a continuous modification that moves toward the desirable strategy.

In this study, we propose a simple Nash equilibrium model and use a simulation method to derive an optimal solution for production decisions by rival firms in oligopoly markets. Aiyoshi and Maki [1] proposed a Nash equilibrium model that applies continuous time replicator dynamics to the analysis of oligopoly markets. Here we consider the game problem of allocating both product and market share, namely resource allocation within the firm to produce their

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