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Original article

Computer generation of random variables with Lindley or Poisson–Lindley distribution via the Lambert *W* function

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Abstract

We provide procedures to generate random variables with Lindley distribution, and also with Poisson–Lindley or zero-truncated Poisson–Lindley distribution, as simple alternatives to the existing algorithms. Our procedures are based on the fact that the quantile functions of these probability distributions can be expressed in closed form in terms of the Lambert *W* function. As a consequence, the extreme order statistics from the above distributions can also be computer generated in a straightforward manner.

AMS Classification: 65C05; 33B30

Keywords: Lindley distribution; Lambert *W* function; Simulation

1. Introduction

In this note, we mainly deal with the computer generation of random data from the Lindley and Poisson–Lindley distributions. More specifically, we derive closed-form expressions for the quantile functions of both probability distributions, which are related to the Lambert *W* function. The particular case corresponding to the zero-truncated Poisson–Lindley distribution will also be considered. The explicit expressions obtained allow us to generate random samples from these probability distributions by means of the inverse transform method.

As it is well-known, the inverse transform method is the most simple procedure to generate random data from a given probability distribution T. This method is used when the quantile function of the random variable T can be expressed in closed form (cf. Fishman [9, pp. 149–156] and also Ross [17, Chapters 4 and 5]). We emphasize that the inverse transform method possesses some advantages over other generation methods. In particular, this method allow to generate samples corresponding to a truncated distribution obtained from T, which sometimes is called restricted sampling, and to this end the closed-form expression of the quantile function of T can be used directly (cf. Fishman [9, p. 152]). Another advantage is that the inverse transform method preserves the monotone relationship between the random variable T and the uniform distribution (cf. Fishman [9, p. 152]); this preservation property is the basis for the use of antithetic variables in order to induce negative correlation between two random variables with the same probability distribution (see also Rubinstein and Kroese [18, Chapter 5]). An additional advantage is that random

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