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Original article

Mass transport with sorption in porous media

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Abstract

Small-scale models in the form of random walks, combining Gaussian jumps, advection by mean flow field and possibly very long sorbing durations, correspond to experimental data in many porous media, in the laboratory and in the field. Within this frame-work, solutes are observed in two phases, which are mobile and immobile. For such random walks, in the hydrodynamic limit, the densities of that phases are linked by a relationship involving a fractional integral. This implies that the total density of tracer evolves according to a fractional variant of Fourier's law.

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1. Introduction

An important feature of porous media is that solutes can be trapped, then released, due to the solid matrix. The idea of an immobile phase for a tracer [5], results in the Mobile-Immobile Model (MIM), assuming exchanges with the mobile fraction, described by first order kinetics. Experimental data [3] showed Break-Through-Curves with very marked heavy tails at the outlet of unsaturated columns with average pore velocity v, better described by the fractal variant of the MIM [2,9,21,23] which is

$$(Id + \lambda I_{0,+}^{1-\gamma})\partial_t P(x, t) = \nabla \cdot (\nabla DP - vP)(x, t), \tag{1}$$

with γ between 0 and 1, $I_{0,+}^{1-\gamma}$ being defined as follows [8,14,16].

Definition 1. The fractional integral of the order of α, computed over $[0, t]$, is $I_{0,+}^{\alpha} f(t) = 1/\Gamma(\alpha) \int_0^t (t - t')^{\alpha-1} f(t')dt'$.

Eq. (1) was proposed at first by Schumer et al. [21], who stressed the interesting asymptotic behavior of the solutions. Then, it was proved [2,9,23] to represent the density $P(x, t)$ of a Brownian Motion subordinated to a time process accounting for immobile periods, distributed by a maximally skewed stable law of exponent γ. Similar equations, with constant coefficients and only the $I_{0,+}^{1-\gamma}\partial_t P(x, t)$ on the left hand side, were also proved to

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