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Original article

Degenerate two-phase compressible immiscible flow in porous media: The case where the density of each phase depends on its own pressure

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Abstract

In this paper, we consider a model of flow of two compressible and immiscible phases in a three-dimensional porous media. The equations are obtained by the conservation of the mass of each phase. This model is treated in its general form with the whole nonlinear terms. We establish an existence result for this model based on new energy estimates to handle the dependence of densities on the corresponding pressure of each phase.

MSC: 35K57; 35K55; 92D30

Keywords: Degenerate system; Nonlinear parabolic system; Compressible flow; Porous media

1. Introduction, assumptions and main results

Many authors studied flows in porous media. The study of the miscible flow models has been investigated in [1,12] and recently in [7–9]. The immiscible and incompressible flows have been treated by many authors [4,6,11,10,13]. For two immiscible compressible flows, we refer to [14,16,5], and recently [15,17,18,2,3].

The immiscible flow models developed by [14–16,3] use the feature of global pressure even if the density of each phase depends on its own pressure, then the context was to assume small capillary pressure so that the densities are assumed to depend on the global pressure, recently and under that context Galusinski and Saad [15] obtained an existence result of solutions.

In this paper, we consider the two compressible immiscible flows model studied in [15], with the difference that we will not use the feature of global pressure in the sense that it enables us to write all models with one pressure variable and one or several saturations with assumption concerns the dependence of densities on a global pressure. The model is treated in its general form under the physical assumption that the density of each phase depends on its own pressure. The mathematical analysis of this model is based on new energy estimates on the pressures. The main idea consists to derive from degenerate estimates on pressure of each phase, which not allowed straight bound on pressures, an estimate

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