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Original Articles

t-Copula generation for control variates

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Abstract

The standard method for generating multi-*t* vectors is simple and convenient but it has the disadvantage that the generated multi-normal and multi-*t* vectors are not similar. For *t*-copula models this destroys much of the variance reduction when using the result of the multi-normal model as external control variate. Therefore we develop a new generation method for multi-*t* vectors. It is based on the polar method and numerical inversion, and generates multi-normal and multi-*t* vectors that are very similar. Numerical experiments with simple functions of the weighted sum of *t*-copula vectors and with pricing European basket options with a *t*-copula model confirm that the obtained variance reduction factors of the new method are high; 2–100 times higher than when using the standard generation method.

Keywords: Monte Carlo simulation; *t*-Copula; Control variate; Polar method

1. Introduction

The practical importance of copula for modeling dependence is well accepted today. For example in financial applications the *t*-copula has become a standard tool, especially for risk quantification. See, e.g. [2,6,9,10,13]. Unfortunately the use of the *t*-copula implies that even simple problems like the calculation of the variance or of tail loss probabilities of weighted sums do not have closed-form solutions. This makes the development of efficient Monte Carlo methods for *t*-copula a practical important task.

Many of the multivariate distributions relevant in practice can be approximated by the multi-normal distribution. If the selected marginals are not too different from normal and the degrees of freedom of the *t*-copula not too small it is also true that *t*-copula models can be approximated by the multi-normal distribution. Many simple problems (like the two mentioned above) have closed form solutions for the multi-normal distribution. It is therefore a natural idea to try to achieve variance reduction by using the approximating multi-normal model as external control variate (CV). But it turns out that this idea leads only to moderate variance reduction factors (for most examples between 2 and 10). This is even true when the distribution of the used *t*-copula is very similar to the multi-normal distribution. How is that possible? The standard approach for generating vectors from the multi-*t* distribution, which is the most common

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