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Original article

Saturation in multivariate simultaneous approximation

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Abstract

In this paper we present a new result on the saturation of sequences of linear operators in a multivariate and simultaneous setting. Specifically, a small o saturation result is obtained for the partial derivatives of the classical Bernstein bivariate operators on the unit simplex. Solutions of boundary value problems for certain partial differential equations of elliptic type play an important role.

Keywords: Saturation; Bivariate Bernstein polynomials; Shape preserving property; Asymptotic formula

1. Introduction

Given $n \in \mathbb{N} = \{1, 2, \ldots\}$, the Bernstein polynomial of order n on the simplex $S = \{(x_1, x_2) \in \mathbb{R}^2; x_1, x_2 \geq 0, x_1 + x_2 \leq 1\}$ is given for $f \in C(S)$ and $x = (x_1, x_2) \in S$ by

$$B_n f(x) = \sum_{k=0}^{n} \sum_{l=0}^{n-k} \binom{n}{k} \binom{n-k}{l} x_1^k x_2^l (1 - x_1 - x_2)^{n-k-l} f\left(\frac{k}{n}, \frac{l}{n}\right), \tag{1}$$

$C(S)$ being the usual space of all continuous functions defined on S endowed with the sup-norm.

Many properties of B_n are very well-known. Among them, if for $x \in S$ we denote $p_i(x) = x_i$, $i = 1, 2$, one has that

$$B_n 1 = 1, \qquad B_n p_i = p_i, \qquad B_n p_i^2 = \frac{1}{n} p_i + \frac{n-1}{n} p_i^2, \quad i = 1, 2, \tag{2}$$

from which one deduces the limit $\|B_n f - f\|_{C(S)} \to 0$ as $n \to \infty$. Further properties on the convergence of this approximation process can be consulted in [7,1]. On the other hand, the determination of the class of functions for which the optimal rate of approximation is achieved, that is, $\|B_n f - f\|_{C(S)} = O(1/n)$ was completely solved in [5]. As for the optimal rate $O(1/n)$ Micchelli [9] had proved earlier that $\|B_n f - f\|_{C(S)} = o(1/n)$ implies that f is linear, and for the local result Ditzian [4] had proved that if $B_n f(x) - f(x) = o_x(1/n)$ for all x in some open ball interior to S, then f is there a solution of the elliptic partial differential equation

$$Du := p_1(1 - p_1) D^{(2,0)} u + p_2(1 - p_2) D^{(0,2)} u - 2 p_1 p_2 D^{(1,1)} u = 0, \tag{3}$$

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