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Effects of a magnetic field on chaotic convection in fluid layer heated from below[☆]

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ABSTRACT

Features of nonlinear dynamics of thermal convection in an electrically conducting fluid layer heated from below and cooled from above subjected to a constant magnetic field are studied theoretically. The Galerkin truncated approximation is employed to derive a low dimensional Lorenz-like model. Numerical analysis was performed to examine the influence of magnetic field on the phase space trajectory of various chaotic regimes. The result indicates that it is possible to suppress or enhance the chaotic convection.

1. Introduction

Chaotic convection in fluid layer has gained considerable attention due to its relevance in a wide range of industrial applications. Chaos was found in a three-dimensional phase space for the Lorenz [1] system arising from the truncation of the classical Rayleigh–Bénard convection model. Convection in a fluid layer heated from below and cooled from above has been studied during the past few decades. Several researchers have realized that chaos can actually be advantageous in various industrial applications such as the production of crystals.

The convective flow is reduced when a magnetic field is imposed on an electrically conducting fluid after the interaction between the imposed magnetic field and the induced electric current. Hence, the presence of a magnetic field is considered to be an effective means for reducing or eliminating these undesired effects in electrically conducting fluids (see Series and Hurle [2]), which represents a method to improve crystal quality.

There are several computational results on the effect of a constant magnetic field. Oreper and Szekely [3,4], Motakef [5], and Kim et al. [6] demonstrated the dissipative influence of the applied magnetic field on the intensity of convection in the melt by using numerical simulation in a vertical Bridgman–Stockbarger configuration. Alboussiere et al. [7] investigated analytically the influence of the cylinder cross-section shape on the core flow structure at large Hartmann number.

Vadasz and Olek [8–12] and Vadasz [13–17] have demonstrated that the transition from steady to chaotic convection in porous media could be recovered from a truncated Galerkin expansion, which yields a system that is identical to the familiar Lorenz equations (see Lorenz [1] and Sparrow [18]). Later, Sheu [19] demonstrated that interface heat transfer metamorphoses the route to chaos and that application of a thermal non-equilibrium model tends to stabilize steady convection. Sheu et al. [20] have shown that stress relaxation tends to accelerate the onset of chaos through the use of an Oldroydian-type constitutive relation. Very recently, Jawdat and Hashim [21] investigated theoretically the significance of internal heating in controlling the transition from steady convection to chaos.

The present work deals with the thermal convective instability of a laterally unbounded horizontal fluid layer heated from below and cooled from above when a constant, vertical magnetic field is applied. The truncated Galerkin approximation was applied to the governing equations to deduce an autonomous system with three ODE, which is important to understand low dimensional dynamics before moving to more complex systems. This system was used to investigate the dynamic behaviour of thermal convection in the fluid layer and justify the influence of magnetic field on the transition from chaos to periodicity and vice versa.

2. Problem formulation

Consider an infinite horizontal electrically conductive fluid layer subject to gravity and heated from below as presented in Fig. 1. A Cartesian co-ordinate system is used such that the vertical axis z is collinear with gravity, i.e., $\hat{e}_g = -\hat{e}_z$. A uniform and constant magnetic field **B** is applied normally to the heated side of the layer. Garandet et al. [22] assumed the magnetic Reynolds number to be small, which implies the insignificance of the induced magnetic field.

A linear relationship between density and temperature is assumed and can be presented as $\rho = \rho_0[1 - \beta(T - T_c)]$ where β represents the thermal expansion coefficient. The time derivative term is not neglected in Darcy's equation for low values of Prandtl number. Besides that, Darcy's law is assumed to govern the fluid flow while the Boussinesq approximation is applied signifying that density variations are effected only for the gravity term in the momentum equation.

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