



Simulation of heat transfer on the peristaltic flow of a Jeffrey-six constant fluid in a diverging tube[☆]

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ABSTRACT

Peristaltic flow of a Jeffrey-six constant fluid in a nonuniform tube is investigated under the assumption of long wavelength and low Reynolds number approximations. The dimensionless quantities are used to simplify momentum and energy equations assuming that fluid physical/rheological properties remain constant. Regular perturbation method is invoked to find an analytical solution for the velocity and temperature field. The variation of pressure rise and frictional forces with the different parameters is also examined numerically. Results are also presented graphically at the end of the article.

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1. Introduction

Peristalsis is the rhythmic contraction of smooth muscles to propel contents through the digestive tract. The fluid is contained within a tube having flexible walls. This phenomenon appears in many biological systems for example smooth muscle tubes such as lower intestine, gastrointestinal tract, cervical canal, female fallopian tube, lymphatic vessels and small blood vessels. Also, peristaltic transport occurs in many practical applications involving biomechanical systems such as roller and finger pumps. Much work on the topic is conducted without heat transfer [1–4]. Few attempts [5–9] have been carried out to discuss peristaltic flows in connection with heat transfer due to its industrial and biological applications. Kothandapani and Srinivas [10] provided the results for the effect of wall properties on the MHD peristaltic flow of a viscous fluid with heat transfer and porous space. Srinivas and Kothandapani have studied the effects of heat transfer on MHD flow of a viscous fluid in an asymmetric channel [11]. The peristaltic flow of viscous fluid in a vertical annulus with heat transfer and MHD effects were carried out by Mekheimer and Abd elmaboud [12]. Vajravelu et al. [13] have studied the influence of heat transfer by considering peristaltic motion in a vertical annulus. Peristaltic flow for third order fluid in the presence of heat transfer analysis has been examined by Nadeem et al. [14]. Due to a variety of applications of non-Newtonian fluids a large number of research papers have been given in the literature few of them are [15–17].

In the present article mathematical description of peristaltic flow of a Jeffrey-six constant fluid in a nonuniform has been investigated. To our knowledge, no attempt has been reported yet to discuss peristaltic flow of a Jeffrey six constant fluid. Momentum and energy

equations for the Jeffrey six constant fluid are formulated considering cylindrical coordinates system. The equations are simplified using the assumptions of long wavelength and low Reynold's number approximation. The simplified non-linear differential equations are then solved by Perturbation technique. The role of different parameters is carefully analyzed.

2. Mathematical formulation

We have considered peristaltic flow of an incompressible Jeffrey-six constant fluid in a nonuniform tube. The flow is generated by sinusoidal wave trains propagating with constant speed c_1 along the walls of the tube. The upper wall of the tube is maintained at temperature \bar{T}_0 and at the centre we have used symmetry condition on temperature. The geometry of the wall surface is defined as

$$\bar{\mathbf{h}} = \mathbf{a}(\bar{Z}) + \mathbf{b} \sin \frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}), \quad (1)$$

where $\mathbf{a}(\bar{Z}) = a_0 + K\bar{Z}$, a_0 is the radius of the inlet, K is the constant whose magnitude depends on the length of the tube, \mathbf{b} is the wave amplitude, λ is the wavelength, c is the propagation velocity and \bar{t} is the time. We are considering the cylindrical coordinate system (\bar{R}, \bar{Z}) , in which \bar{Z} -axis lies along the center line of the tube and \bar{R} is transverse to it (Fig. 1).

The governing equations in the fixed frame for an incompressible flow are given as

$$\frac{\partial \bar{U}}{\partial \bar{R}} + \frac{\bar{U}}{\bar{R}} + \frac{\partial \bar{W}}{\partial \bar{Z}} = 0, \quad (2)$$

$$\rho \left(\frac{\partial}{\partial \bar{t}} + \bar{U} \frac{\partial}{\partial \bar{R}} + \bar{W} \frac{\partial}{\partial \bar{Z}} \right) \bar{U} = - \frac{\partial \bar{p}}{\partial \bar{R}} + \frac{1}{\bar{R}} \frac{\partial}{\partial \bar{R}} (\bar{R} \bar{\tau}_{\bar{R}\bar{R}}) + \frac{\partial}{\partial \bar{Z}} (\bar{\tau}_{\bar{R}\bar{Z}}) + \frac{\bar{\tau}_{\bar{\theta}\bar{\theta}}}{\bar{R}}, \quad (3)$$

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