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Diffusion in homogeneous layer with cylindrical hole partially filled by diffusing substance ☆

Stanisław J. Matysiak [a,*], Radosław Mieszkowski [a], Dariusz M. Perkowski [b]

[a] Institute of Hydrogeology and Engineering Geology, Faculty of Geology, University of Warsaw, Al. Żwirki i Wigury 93, 02-089 Warsaw, Poland
[b] Faculty of Mechanical Engineering, Białystok University of Technology, 15-351 Białystok, Wiejska St., 45C, Poland

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ABSTRACT

The paper deals with the stationary problems of diffusion in a homogeneous isotropic layer with a cylindrical hole. The hole is partially filled by diffusing substance. The considered problem is described within the framework of the Fick's law and the integral Weber–Orr transform method is applied to solve it. The obtained exact results expressed by integrals will be presented in the form of figures.

1. Introduction

Diffusion is the process which plays important role in the environment protection, engineering geology, hydrogeology and applied geophysics [1–12], as well as in the chemical engineering [13–15]. In an aspect of the environment protection one of the utilization methods of toxic and radioactive wastes is storing their wastes in underground mining excavations (drifts, mine chambers, and bore-holes). Wastes are transported in special containers, in which leakproofnesses are rather not guaranteed. In this situation a prognosis of the size and intensity of environment contamination is advisable. The migration of wastes takes place on the way of flow and diffusion. The process of flow is observed in porous media, however when a rock mass is built from compacted soils, a pollution will be migrated as random molecular motions.

This paper deals with the stationary diffusion problem in a homogeneous, isotropic layer with vertically located, cylindrical hole. The diffusing substance is stored in some lower part of the hole and the pollution migrates by lateral surface of this hole part, while the remaining upper part of the hole as well as the upper and lower surfaces of the layer are assumed to be free from the diffusing substance. The considered problem is axisymmetric and independent on time. The aim of this paper is to find the distribution of substance concentration and its gradient in the layer. The above formulated problem and its solution can be some theoretical basis for adequate geological methods to qualifications of a direction, a range and a velocity migrating indicatory solution (for example: the conductometric method, the isotopic method). The main idea of these methods consists of an introduction to the bore-hole an indicatory substance (e.g.: salt solutions or radioactive isotopes) and after that measurement changes of the electric resistance or the radioactive coefficient, respectively.

2. Formulation and solution of the problem

Consider a homogeneous isotropic layer with the thickness h resting on an impermeable substrate. The cylindrical hole of radius a is located perpendicular to the boundary planes of the layer and is partially filled by a diffusing substance. The problem will be described by using the cylindrical coordinates (r, φ, z) such that the axis Oz is the axis of symmetry of the hole and the upper surface of substance is located on the plane $z=0$, see Fig. 1. Let $z=h_1$ and $z=-h_2$, $r>a$ be the lower and the upper boundary planes of the layer, so $h=h_1+h_2$. It will be assumed that the lower and upper boundaries are free from the diffusing substance (zero concentration). The part of lateral hole surface is kept at a given constant concentration c_0 for $0<z<h_1$, the remaining part for $-h_2<z<0$ at zero concentration. The considered problem is stationary (independent on time) and axisymmetric (independent on the variable φ). Divide the considered layer with hole on two sublayers: $S_1=\{(r,z);\, r>a,\, -h_2<z<0\}$ and $S_2=\{(r,z);\, r>a,\, 0<z<h_1\}$. This partition is caused by a mathematical reason (further mathematical calculations). Let $c\,(r, z)$ denote the concentration of diffusing substance at the point (r, z) and $c^{(i)}$ be the concentration in sublayer S_i, so $c(r,z)\equiv c^{(i)}(r,z)$, for $(r,z)\in S_i$, $i=1, 2$.

The considered problem is described by the equations for unknown concentrations $c^{(i)}(r, z)$ in the form [16]:

$$\frac{\partial^2 c^{(i)}(r,z)}{\partial r^2}+\frac{1}{r}\frac{\partial c^{(i)}(r,z)}{\partial r}+\frac{\partial^2 c^{(i)}(r,z)}{\partial z^2}=0, \quad \text{for} \quad (r,z)\in S_i, \ i=1,2, \tag{1}$$

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* Corresponding author.
E-mail address: s.j.matysiak@uw.edu.pl (S.J. Matysiak).

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