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Thin film flow over a heated nonlinear stretching sheet in presence of uniform transverse magnetic field $\overset{\backsim}{\asymp}$

B.S. Dandapat ^{a,*}, S.K. Singh ^b

^a Department of Mathematics, Sikkim Manipal Institute of Technology, Majitar, East sikkim 737 132, Sikkim, India

^b Physics and Applied Mathematics Unit, Indian Statistical Institute, 203-B. T. Road, Calcutta-700 108, India

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ABSTRACT

The unsteady flow of a thin viscous liquid film over a heated horizontal stretching surface permitted by uniform transverse magnetic field is studied by considering the stretching velocity and the temperature distribution in their general functional forms. An evolution equation for the film thickness is derived using long-wave approximation theory of thin liquid film and this nonlinear PDE is solved numerically for some representative values of non-dimensional parameters. It is observed that the magnetic field resists the film thinning process for all types of velocity and temperature distribution. But thermocapillarity enhances the film thinning rate even in presence of magnetic field. Further, effect of Marangoni and Prandtl numbers are explored in presence of magnetic field. Physical explanations are provided to understand the different effects on film thinning.

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1. Introduction

The flow and heat transfer phenomena of a viscous liquid over a stretching surface has applications in a number of technological processes such as metal and polymer extrusion, continuous casting, drawing of plastic sheets, cable coating etc. In these processes, the quality of the final product very much depends on the rate of heat and mass transfer of the system. Crane [1] first studied the steady twodimensional boundary layer flow due to the stretching of a flat elastic sheet. Due to its practical applications, the stretching sheet flow problem has attracted several researchers for the last three decades and is extensively studied to understand the same, along with either the sole effects of rotation, heat and mass transfer, chemical reaction, MHD, suction/injection, different non-Newtonian fluid or different possible combinations of these above effects ([2–14]). Needless to say that in all these studies, the boundary layer equation is considered and the boundary conditions are prescribed at the sheet and on the fluid outside the boundary layer tends to infinity. The imposition of similarity transformation reduces the system to a set of ODEs, which are then solved either analytically or numerically. To the best of our knowledge, the study of unsteady flow due to the stretching of a sheet has not yet received adequate attention when the film and the boundary layer thickness coincide, despite the extensive research made on this flow problem since the last three decades. A few research works have been carried out by considering unsteady stretching of a thin liquid film placed over the sheet. In these studies

E-mail address: bsdandapat@rediffmail.com (B.S. Dandapat).

also a special type of transformation is used to express the boundarylayer equations into their similarity form and solved either numerically or analytically (Wang [15], Andersson et al. [16], Chen [17] and Dandapat et al. ([18,19]) and many others). In these investigations crude assumptions were that the initial liquid distribution over the stretching sheet is uniform and the film height varies only with time but not along the space. In general film thickness varies with time and space during stretching. Recently, Dandapat et al. [20] and Dandapat and Maity [21] have studied the development of thin liquid film over a stretching sheet with non-planar film free surface at the onset of stretching. In these studies they have assumed that the boundary laver covers the entire depth of the fluid and the Navier-Stokes equations are solved analytically using the matched asymptotic method and the method of characteristic to predict the variation of film thickness with space and time. But the tacit assumption in these studies was that the sheet is stretched linearly. Later, nonlinear stretching was studied by Santra and Dandapat ([22,23]) for nonplanar film surface and solved numerically. In [23], they have found that the thermocapillary effects are responsible in shaping the film thickness. Further the thermocapillary effects are more pronounced for lower values of Prandtl number and Biot number.

In the present article, we have tried to explore the effect of transverse magnetic field and thermocapillarity on film thinning process and temperature development across the interface for linear/ nonlinear stretching velocity and temperature distribution in stretching sheet. It is to be mentioned here that the development of conducting liquid film has equally important applications in the industry. The remaining parts of the paper are arranged as follows. In the next section, governing equations are expressed into their dimensionless form and are solved by using long wave theory to

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^{*} Corresponding author.

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