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The application of differential transformation method to nonlinear equations arising in heat transfer ☆

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ABSTRACT

In this paper two nonlinear heat transfer problems were solved by considering variable specific heat coefficient. The calculations are carried out by using differential transformation method (DTM) which is a semi-numerical–analytical solution technique. By using DTM, the nonlinear constrained governing equations are reduced to recurrence relations and related initial conditions are transformed into a set of algebraic equations. The principle of differential transformation is briefly introduced, and then applied for the aforementioned problems. The solutions are subsequently solved by a process of inverse transformation. The current results are then compared with those derived from the variational iteration method (VIM), homotopy perturbation method (HPM), perturbation method (PM) and the exact solutions in order to verify the accuracy of the proposed method. The findings reveal that the DTM can achieve more suitable results in predicting the solution of such problems.

1. Introduction

Ordinary differential equations are generally solved using integral transformation methods such as the Laplace or Fourier transform. These methods transform the ordinary differential equation into a corresponding algebraic equation. However, applying these integral transformation methods to the solution of nonlinear ordinary differential problems is problematic. The differential transformation method (DTM) is a semi-numerical–analytical method for solving problems of this type. DTM which is based on the Taylor series expansion was first proposed by Zhou [1] in 1986 for the solution of linear and nonlinear initial value problems that appear in electrical circuits. This method obtains a solution in the form of a polynomial. Indeed, this fact can be seen in the next section where the concept of differential transform is briefly described. Later, this method has been used to obtain numerical and analytical solutions of ordinary differential equations, partial differential equations, difference equations, and integro differential equations [2–10]. There are recent studies on the application of DTM to the heat transfer problems in literature. Bert [11] applied DTM to a steady-state heat transfer in a triangular-profile fin with constant properties. Kuo [12] applied DTM to investigate the temperature field associated with the Falkner–Skan boundary-layer problem. Chu and Chen [13] applied a hybrid method of differential transform and finite difference method to solve a transient heat conduction problem which had complex nonlinear terms. Chu and Lo [14] presented a hybrid differential transformation-finite difference method to analyze nonlinear transient heat conduction problems. Lo and Chen [15] proposed an alternative numerical method to investigate hyperbolic heat conduction problems using the hybrid differential transfer/control-volume method. Joneidi et al. [16] studied analytical solution of fin efficiency of convective straight fins with temperature-dependent thermal conductivity by DTM. Jang et al. [17] investigated and characterized a two-dimensional thermal conductive boundary value problem with discontinuous boundary and initial conditions. Rashidi et al. [18] applied DTM to find the analytic solution for the problem of mixed convection about an inclined flat plate embedded in a porous medium. Rashidi and Mohimanian Pour [19] applied DTM to steady flow over a rotating disk in porous medium with heat transfer.

The pursuit of analytical solutions for the heat conduction equations are of intrinsic scientific interest. In this paper, the basic idea of DTM is described, then it is applied to nonlinear lumped heat conduction system with convection and to nonlinear convective–radiative cooling equation. Moreover, it was assumed that the specific heat is a linear function of temperature, and we have made a comparison with the variational iteration method (VIM), homotopy perturbation method (HPM), perturbation method (PM) and the exact solutions in order to verify the accuracy of the proposed method.

2. Fundamental of differential transformation method

Let $x(t)$ be analytic in a domain D and let $t=t_i$ represents any point in D. The function $x(t)$ is then represented by one power series whose

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