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Global stability for thermal convection in a couple-stress fluid☆

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ABSTRACT

We show that the global nonlinear stability threshold for convection in a couple-stress fluid is exactly the same as the linear instability boundary. This optimal result is important because it shows that linearized instability theory has captured completely the physics of the onset of convection. It is also found that the couple-stress fluid is thermally more stable than the ordinary viscous fluid and then the effect of couple stress parameter on the onset of convection is also analyzed.

1. Introduction

Convectional hydrodynamic stability theory is mainly concerned with the determination of critical values of Rayleigh number, demarcating a region of stability from that of instability. The potentials of the linear theory of stability and of the energy method are complementary to each other in the sense that the linear theory gives conditions under which hydrodynamic systems are definitely unstable. It cannot with certainty conclude stability. On the other hand, energy theory gives conditions under which hydrodynamic systems are definitely stable. It cannot with certainty conclude instability. Suffering from its basic assumptions, the validity of the linearized stability theory becomes questionable. Hence, the nonlinear approach becomes inevitable to investigate the effects of finite disturbances.

The oldest method of nonlinear stability analysis which can deal with finite disturbances is the energy method, originated by Reynolds [1], Orr [2] and then later Serrin [3] and Joseph [4–6] reformulated the energy method. Despite the success of this classical energy method in several stability problems, there is some skepticism about its ongoing indiscriminate use. Situations have been encountered, for example, in magnetic Bénard problem (Rionero [7], Galdi [8]) and in the rotating Bénard problem (Galdi and Straughan [9]), where the classical energy theory did not produce expected results. Rapid improvements of the classical energy theory have been made in recent years (Galdi and Padula [10], Straughan [11]) and the Lyapunov direct method employed by Galdi [8], Galdi and Straughan [9], Rionero and Mulone [12], Mulone and Rionero [13], Galdi and Padula [10], Qin and Kaloni [14] appears to have been the most successful one. It is now generally believed that this generalized energy method is definitely superior to the classical energy method. A nonlinear stability analysis of fluids by using generalized energy stability theory has been considered by many authors (Guo et al. [15], Guo and Kaloni [16], Straughan and Walker [17], Kaloni and Qiao [18–20], Straughan [21], Payne and Straughan [22], Straughan [23]). Recently, Sunil and Mahajan [24–29] have studied the nonlinear stability analysis for magnetized ferrofluid by using energy method. They found that the nonlinear critical stability magnetic thermal Rayleigh number does not coincide with that of the linear instability analysis, and thus indicates that the subcritical instabilities are possible. However, it is noted that, in the case of non-ferrofluid, global nonlinear stability Rayleigh number is exactly the same as that for linear instability.

There are a lot of analyses of performance and experiment in the couple-stress lubricant. Stokes [30] proposed a simplest theory called the Stokes micro-continuum theory and which could be used for the simulation of the couple-stress fluid. This kind of couple-stress model is intended to take account of the particle-size effects, and it is also very useful in the scientific and engineering applications. Ramanaiah [31] applied the couple-stress fluid model to analyze the long slider bearing. Gupta and Sharma [32] also used the couple-stress fluid model to carry out a hydrostatic thrust bearing. Shehawey and Mekheimer [33] applied the couple-stress model to analyze the peristalsis problem for its relative mathematical simplicity. Das [34] proposed the analysis of elastohydrodynamic theory of line contacts. Das [35] studied the slider bearing lubricated with couple-stress fluids in magnetic field and observed that both the values of the maximum load capacity and the corresponding inlet–outlet film ratio depend on couple stress, magnetic parameters and the shape of bearings. Abdallah and Lotfi [36] proposed an efficient numerical scheme to solve the direct lubrication problem for journal bearing lubricated with couple-stress fluids, which consists of the modified Reynolds equation, the film thickness equation, and the boundary for the pressure field. Hsu et al. [37] studied the short journal bearings lubricated with the non-Newtonian fluid which is combined the effects of couple stresses and surface roughness. It was found that the

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