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Bio-thermal convection induced by two different species of microorganisms ☆

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ABSTRACT

This paper develops a theory of bio-thermal convection in a suspension that contains two species of microorganisms exhibiting different taxes, gyrotactic and oxytactic microorganisms. The developed theory is applied to investigating the onset of bio-thermal convection in such a suspension occupying a horizontal layer of finite depth. A linear stability analysis is utilized to derive the equations for the amplitudes of disturbances. The obtained eigenvalue problem is solved by the Galerkin method. The case of non-oscillatory instability in a layer with a rigid lower boundary and a stress-free upper boundary is investigated. The resulting eigenvalue equation relates three Rayleigh numbers, the traditional Rayleigh number (Ra) and two bioconvection Rayleigh numbers, one for gyrotactic ($\mathrm{Rb_g}$) and one for oxytactic ($\mathrm{Rb_o}$) microorganisms. The neutral stability boundary is presented in the form of a diagram showing that boundary in the (Ra, $\mathrm{Ra_g}$) plane for different values of $\mathrm{Ra_o}$.

1. Introduction

Recently there has been increased interest in research addressing biononvection, a macroscopic convective fluid motion induced by upswimming of motile microorganisms [1–5]. In [6] the effect of bioconvection on the dynamics of plankton population was analyzed. Recently, Kuznetsov [7] suggested using motile microorganisms to induce mixing and prevent nanoparticle agglomeration in nanofluids.

The purpose of this paper is to introduce the theory of bio-thermal convection induced by the presence of two species of microorganisms exhibiting different taxes, gyrotactic and oxytactic microorganisms. This system is fundamentally interesting and important because the presence of two species exhibiting different taxes would give an experimentalist an additional control of the system's behavior. This paper investigates the onset of instability in such a suspension occupying a horizontal layer of finite depth.

2. Governing equations

A water-based dilute suspension containing both gyrotactic and oxytactic microorganisms is considered. The suspension occupies a horizontal layer of depth H. The governing equations are based on the theory of bioconvection in suspensions of gyrotactic microorganisms developed in [8–10], the theory of bioconvection in suspensions of oxytactic microorganisms developed in [11,12], and the theory of bio-thermal convection developed in [13]. The dimensionless governing equations are

$$\nabla \cdot \mathbf{U} = 0 \tag{1}$$

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = -\nabla p + \nabla^2\mathbf{U} - \mathrm{Rm}\hat{\mathbf{k}} + \mathrm{Ra}T\hat{\mathbf{k}} - \frac{\mathrm{Rb}_g}{\mathrm{Lb}_g}n_g\hat{\mathbf{k}} - \frac{\mathrm{Rb}_o}{\mathrm{Lb}_o}n_o\hat{\mathbf{k}} \tag{2}$$

$$\frac{\partial T}{\partial t} + \mathbf{U}\cdot\nabla T = \nabla^2 T \tag{3}$$

$$\frac{\partial n_g}{\partial t} = -\nabla\cdot\left(n_g\mathbf{U} + n_g\frac{\mathrm{Pe}_g}{\mathrm{Lb}_g}\hat{\mathbf{p}} - \frac{1}{\mathrm{Lb}_g}\nabla n_g\right) \tag{4}$$

$$\frac{\partial n_o}{\partial t} = -\nabla\cdot\left(n_o\mathbf{U} + n_o\mathbf{V}_o - \frac{1}{\mathrm{Lb}_o}\nabla n_o\right) \tag{5}$$

$$\frac{\partial C}{\partial t} + \mathbf{U}\cdot\nabla C = \frac{1}{\mathrm{Le}}\nabla^2 C - \hat{\beta}n_o \tag{6}$$

Dimensionless variables in Eqs. (1)–(6) are introduced as follows:

$$(x,y,z) = (x^*,y^*,z^*)/H, \quad t = t^*\alpha_f/H^2, \ \mathbf{U} = (u,v,w) = (u^*,v^*,w^*)H/\alpha_f \tag{7}$$

$$p = p^*H^2/\mu\alpha_f, \ \ T = \frac{T^*-T_c^*}{T_h^*-T_c^*}, \ \ n_g = n_g^*/n_{g,av}^*, \ \ n_o = n_o^*/n_{o,av}^* \tag{8}$$

where (x^*,y^*,z^*) are Cartesian coordinates (z^* is the vertically upward coordinate); t^* is the time; α_f is the thermal diffusivity of water, $k/(\rho c)_f$; $\mathbf{U}^* = (u^*,v^*,w^*)$ is the fluid velocity; p^* is the pressure; μ is the viscosity of water; T^* is the temperature; T_c^* is the reference temperature (the temperature of the upper wall); T_h^* is the temperature of the lower wall; n_g^* is the concentration of gyrotactic microorganisms; $n_{g,av}^*$ is the average concentration of gyrotactic

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