International Communications in Heat and Mass Transfer 38 (2011) 565–571

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

The UNIT algorithm for solving one-dimensional convection-diffusion problems via integral transforms☆

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ARTICLE INFO

Available online 13 January 2011

Keywords:
Generalized Integral Transform Technique
Hybrid methods
Convection-diffusion
Non-linear problems

ABSTRACT

A unified approach for solving convection-diffusion problems using the Generalized Integral Transform Technique (GITT) was advanced and coined as the UNIT (UNified Integral Transforms) algorithm, as implied by the acronym. The unified manner through which problems are tackled in the UNIT framework allows users that are less familiar with the GITT to employ the technique for solving a variety of partial-differential problems. This paper consolidates this approach in solving general transient one-dimensional problems. Different integration alternatives for calculating coefficients arising from integral transformation are discussed. Besides presenting the proposed algorithm, aspects related to computational implementation are also explored. Finally, benchmark results of different types of problems are calculated with a UNIT-based implementation and compared with previously obtained results.

1. Introduction

The Generalized Integral Transform Technique (GITT) [1–9] has been demonstrated to be a powerful tool for solving a variety of convection-diffusion problems. This technique is based on using orthogonal eigenfunctions expansions for expressing the unknown dependent variables; however, different from the Classical Integral Transform Technique [10], the transformation of the original problem needs not lead to a decoupled system, making the method applicable to a large number of problems.

The resulting transformed system is usually composed of a set of ODEs, which can be readily solved by well-established numerical routines that enable user-prescribed accuracy control. This, together with the analytical nature of this technique, allows for better global error control while compared to traditional domain discretization methods. The main drawback usually associated with the GITT is that a notable amount of analytical work can be required; nevertheless, this problem can be circumvented by the usage of symbolical computation [7,9,11].

Some of the most recent applications of the Generalized Integral Transform Technique include, convective heat transfer in flows within wavy walls [12], hyperbolic heat conduction problems [13], conjugated conduction-convection problems [14], transient diffusion in heterogeneous media [15], heat and mass transfer in adsorption [16], atmospheric pollutant dispersion [17] and dispersion in rivers and channels [18], heat transfer in MHD [19], applications to irregular geometries [20], solution of the Navier–Stokes equations [21] and the boundary layer equations [22], stability analysis in natural convection [23], among others.

A characteristic aspect of all previous integral transform studies is that the solution strategy used in each work is tailored to the specific application. Although this can be very effective for individual analyses, when applying an existing solution strategy to a different problem, several adaptations could be necessary. This becomes particularly difficult for users not so well-versed with the Generalized Integral Transform Technique. Under this scenario, this contribution proposes a unified algorithm for solving virtually any convection-diffusion problem via Integral Transforms. This approach, named UNIT (UNified Integral Transforms), has the potential to enable a substantially greater number of users to apply the GITT for solving a variety of problems. Preliminary developments regarding the ideas behind this algorithm lead to two recent studies [24,25]. A first application of a preliminary UNIT algorithm was also used in the solution of the bio-heat equation [26]. After these studies, the current work offers a consolidation of the unified integral transform approach, introducing the complete version of the one-dimensional UNIT algorithm in a formal mathematical fashion. In addition, simulation results of different test-problems are presented, providing a verification of the algorithm against previous literature results.

☆ Communicated by W. J. Minkowycz.

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0735-1933/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2010.12.036