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Two new differential equations of turbulent dissipation rate and apparent viscosity for non-newtonian fluids ☆

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ABSTRACT

A new equation for the dissipation rate of turbulent kinetic energy is derived exactly in conservative form for a Generalized Newtonian Fluid (GNF). The transport equations for mass, momentum, and turbulent kinetic energy are written along to the transport equation for the shear rate. A new transport equation for the apparent viscosity is derived assuming the viscosity as dependent only on the shear rate. The assumption is of incompressible two-dimensional GNF flow.

1. Introduction

Non-Newtonian fluids are present in several industrial applications and biological problems, like blood flow. Literature presents many theoretical solutions and numerical simulations in laminar flow, including two papers published by the first author more than 30 years ago [1,2].

Few numerical investigations dealt with turbulent flow of pseudo-plastic fluids (shear-thinning fluid) because of the lack of models with one or two point closure and, for this reason, some investigators performed DNS (Direct Numerical Simulation). Rudman and Blackburn [3] used the Spectral Element-Fourier Method (SEM) in a duct flow and compared the DNS results of a power law fluid with small consistency index and of a Herschel-Bulkley fluid with experimental data [4]. Dimitropoulos [5,6] carried out DNS for a polymeric solution using FENE-P and *Giesekus* models with spectral approximation and semi-implicit algorithm to predict the drag reduction. New results on Reynolds stresses and pressure are presented in [6], where the convergence of the pseudo-spectral algorithm is discussed. A non-refined mesh and a high artificial viscosity are introduced to stabilize the algorithm. The FENE-P model is used in [7] for a DNS one-dimensional approach to explain the phenomenon of drag reduction. A turbulent model for a non-Newtonian power law fluid is developed in [8], in analogy to the turbulent viscosity, determining the temperature distribution for soybean milk flowing inside a tubular heat exchanger.

Turbulent flow of a non-Newtonian fluid is important also in medical field. A model to predict the turbulent flow of a power-law fluid in a bio-reactor for anaerobic digestion is developed in [9] with the classical k-ε model and the power-law viscosity. The k-ε equations are derived in [10,11] for power-law and Herschel–Bulkley fluids using the apparent viscosity of a non-Newtonian fluid in the RANS equations of a Newtonian fluid, but the agreement is not good enough. The introduction of the third invariant of the rate of deformation tensor in the viscosity contributes to an increase of viscous diffusion and dissipation rate in the turbulent kinetic energy confirming the dependence of the viscosity on the second invariant of the rate of deformation tensor in a 2D flow [12]. The Generalized Newtonian Fluid (GNF) constitutive equation is applied to a *Bird-Carreau* fluid in order to derive a k–ε model for the equations of the Reynolds stresses tensor, turbulent kinetic energy and dissipation rate [13]. The viscosity is dependent on the invariants of the rate of deformation tensor, shear-rate and strain-rate. An algebraic equation is proposed to correlate the instantaneous viscosity to the dissipation rate while average viscosity and dissipation rate are correlated with a normal logarithmic probability distribution of the dissipation rate. The final equation of dissipation rate is written in non conservative form because two derivatives are present, one for the dissipation rate itself and the other for the average dynamic viscosity.

Direct Numerical Simulation of viscoelastic fluids in turbulent channel flow is carried out using the FENE-P model to find relationships between flow and fluid rheological parameters [14]. Three different regimes of drag reduction, namely low, high and medium are identified proposing mathematical expressions for the eddy viscosity in the three regimes. A procedure based on the DNS predictions of the budgets of momentum and viscoelastic shear stress is developed to evaluate the mean velocity profile. A RANS model is employed using the FENE-P constitutive relationship to describe the rheology of polymer-induced turbulent drag reduction [15]. Correlations among flow and polymer conformation variables are identified by analyzing recent DNS results of dilute polymer solutions.

The present work is aimed to derive the equation of the turbulent dissipation rate in a conservative form for an incompressible GNF in 2D (two-dimensional) flow. Viscosity is depending on shear-rate, as shown

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