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# Note on the effect of thermal radiation in the linearized Rosseland approximation on the heat transfer characteristics of various boundary layer flows ☆

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ABSTRACT

In the latter years the title problem has been examined in a large number of research papers. The present Note emphasizes, however, that the effect of thermal radiation in the linearized Rosseland approximation is quite trivial, both physically and computationally. Namely, it always reduces to a simple rescaling of the Prandtl number by a factor involving the radiation parameter. This implies that a comprehensive study of the Prandtl-number dependence without thermal radiation effects represents per se a detailed study of the radiation effects, too. In other words, the solution of the radiation problem for optically thick media in the linearized Rosseland approximation does not require any additional numerical or analytical effort compared to the same problem without radiation, making in this respect dozens of papers superfluous.


The Rosseland approximation [1] applies to optically thick media and gives the net radiation heat flux $q_r[Wm^{-2}]$ by the expression

$$q_r = -\frac{4}{3a_R} grad(e_b). \tag{1}$$

Here $a_R[m^{-1}]$ is the Rosseland mean spectral absorption coefficient and $e_b[W\,m^{-2}]$ the blackbody emissive power which is given in terms of the absolute temperature $T$ by the Stefan-Boltzmann radiation law $e_b = \sigma_{SB}\,T^4$, with the Stefan-Boltzmann constant $\sigma_{SB} = 5.6697 \cdot 10^{-8}\,W\,m^{-2}K^{-4}$.

For a plane boundary layer flow over a hot surface, Eq. (1) of the net radiation heat flux absorbed in the fluid reduces to

$$q_r = -\frac{16\sigma_{SB}}{3a_R} T^3 \frac{dT}{dy} \tag{2}$$

where $y$ denotes the coordinate along the normal to the wall.

The physical and mathematical advantage of the Rosseland formula (2) consists of the fact that it can be combined with Fourier's second law of conduction to an *effective conduction-radiation flux* in the form

$$q_{eff} = -\left(k + \frac{16\sigma_{SB}}{3a_R} T^3\right) \frac{\partial T}{\partial y} \equiv -k_{eff}(T) \frac{\partial T}{\partial y} \tag{3}$$

where the *effective thermal conductivity* has been defined as

$$k_{eff}(T) = k(T) + \frac{16\sigma_{SB}}{3a_R} T^3. \tag{4}$$

In addition to the explicit dependence of $k_{eff}$ on $T^3$, in Eq. (4) the "usual" thermal conductivity $k[W\,m^{-1}K^{-1}]$ is also a function of $T$.

Bearing in mind the above considerations, the steady energy balance equation including the net contribution of the radiation emitted from the hot wall and absorbed in the colder fluid, takes the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left[k_{eff}(T)\frac{\partial T}{\partial y}\right] + other\ terms \tag{5}$$

where in *"other terms"* the possible contributions of volumetric heat sources and/or sinks, of viscous dissipation, of the pressure work, of the Joule heating etc. have been subsumed.

Eq. (5) is highly nonlinear in $T$ and has attracted during the latter fifty years a vast research interest in various contexts of the boundary layer heat transfer, both of Newtonian and non-Newtonian fluids (see e.g. Viskanta and Grosh [2], Cess [3], Sparrow and Cess [4], Hossain et al. [5], Hossain et al. [6], Ghaly [7], Pop et al. [8], Kumari and Nath [9], Jat and Chaudhary [10], to name only a few of them).

A massive simplification of the energy Eq. (5) can be achieved when the temperature gradients within the flow are small. In such cases the "usual" thermal conductivity $k$ can be considered as a constant and the Rosseland formula (2) can be linearized about the

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