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Multi-parametric solution-path algorithm for instance-weighted support vector machines

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Abstract An *instance-weighted* variant of the support vector machine (SVM) has attracted considerable attention recently since they are useful in various machine learning tasks such as non-stationary data analysis, heteroscedastic data modeling, transfer learning, learning to rank, and transduction. An important challenge in these scenarios is to overcome the computational bottleneck—instance weights often change dynamically or adaptively, and thus the weighted SVM solutions must be repeatedly computed. In this paper, we develop an algorithm that can efficiently and exactly update the weighted SVM solutions for arbitrary change of instance weights. Technically, this contribution can be regarded as an extension of the conventional *solution-path* algorithm for a single regularization parameter to multiple instance-weight parameters. However, this extension gives rise to a significant problem that *breakpoints* (at which the solution path turns) have to be identified in high-dimensional space. To facilitate this, we introduce a parametric representation of instance weights. We also provide a geometric interpretation in weight space using a notion of *critical region*: a polyhedron in which the current affine solution remains to be optimal. Then we find break-

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