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Optimal control as a graphical model inference problem

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Abstract We reformulate a class of non-linear stochastic optimal control problems introduced by Todorov (in Advances in Neural Information Processing Systems, vol. 19, pp. 1369–1376, 2007) as a Kullback-Leibler (KL) minimization problem. As a result, the optimal control computation reduces to an inference computation and approximate inference methods can be applied to efficiently compute approximate optimal controls. We show how this KL control theory contains the path integral control method as a special case. We provide an example of a block stacking task and a multi-agent cooperative game where we demonstrate how approximate inference can be successfully applied to instances that are too complex for exact computation. We discuss the relation of the KL control approach to other inference approaches to control.

Keywords Optimal control · Uncontrolled dynamics · Kullback-Leibler divergence · Graphical model · Approximate inference · Cluster variation method · Belief propagation

1 Introduction

Stochastic optimal control theory deals with the problem to compute an optimal set of actions to attain some future goal. With each action and each state a cost is associated and the aim is to minimize the total future cost. Examples are found in many contexts such as motor

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