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Almost optimal estimates for approximation and learning by radial basis function networks

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Abstract This paper quantifies the approximation capability of radial basis function networks (RBFNs) and their applications in machine learning theory. The target is to deduce almost optimal rates of approximation and learning by RBFNs. For approximation, we show that for large classes of functions, the convergence rate of approximation by RBFNs is not slower than that of multivariate algebraic polynomials. For learning, we prove that, using the classical empirical risk minimization, the RBFNs estimator can theoretically realize the almost optimal learning rate. The obtained results underlie the successful application of RBFNs in various machine learning problems.

Keywords Learning theory · Approximation theory · Radial basis function networks · Rate of convergence

1 Introduction

In physical or biological systems, engineering applications, financial studies, and many other fields, only a finite data set $(x_i, y_i)_{i=1}^m$ be obtained. Learning means synthesizing a function that best represents the relation between the inputs and the corresponding outputs. A learning system is normally developed for defining the function and yielding an estimator. The learning system comprises a hypothesis space, a family of parameterized functions that regulate the forms and properties of the estimator to be found, and a learning strategy or learning algorithm that numerically yields the parameters of the estimator. The central question of learning is and will always be: how well does the synthesized function generalized to reflect the reality that the given samples purport to show us.

The analysis of a learning system can be regarded as studying approximation capability of the hypothesis space and efficiency of the learning strategy. From the point of view

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