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Stratified Generalized Procrustes Analysis

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Abstract Generalized procrustes analysis computes the best set of transformations that relate matched shape data. In shape analysis the transformations are usually chosen as similarities, while in general statistical data analysis other types of transformation groups such as the affine group may be used. Generalized procrustes analysis has a nonlinear and nonconvex formulation. The classical approach alternates the computation of a so-called reference shape and the computation of transformations relating this reference shape to each shape datum in turn.

We propose the stratified approach to generalized procrustes analysis. It first uses the affine transformation group to analyze the data and then upgrades the solution to the sought after group, whether Euclidean or similarity. We derive a convex formulation for each of these two steps, and efficient practical algorithms that gracefully handle missing data (incomplete shapes).

Extensive experimental results show that our approaches perform well on simulated and real data. In particular our closed-form solution gives very accurate results for generalized procrustes analysis of Euclidean data.

Keywords Shape · Registration · Procrustes · Theseus · Generalized

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1 Introduction

In many different problems, data analysis requires one to first compensate for a global transformation between the different datasets or *shape data*. This is known as *procrustes analysis* in the statistics and shape analysis literature (Dryden and Mardia 1998; Gower and Dijksterhuis 2004). Each shape datum is a set of points. This analysis is called *generalized procrustes analysis* when more than two shape data are to be registered. In this problem, one global transformation per observed shape has to be computed, so that the shapes are mapped to a common coordinate frame whereby they look as 'similar' as possible. This process is called also *rigid registration*. For example, it is typical in landmark-based 2D and 3D shape analysis to compensate for a similarity (rotation, translation and scale factor) so as to normalize the shapes to be further analyzed. The global transformation is usually modeled by one of the groups of Euclidean transformations ($\frac{1}{2}d(d+1)$ degrees of freedom), similarity transformations ($\frac{1}{2}d(d+1)+1$ degrees of freedom) and affine transformations ($d(d+1)$ degrees of freedom), with d the dimension of the shape data to be analyzed. The classical approach to generalized procrustes analysis is to select one of the shapes as a *reference shape*, and register each of the other shapes to the reference in turn. It is common to then alternate a re-estimation of the reference shape, as the average of the registered shapes, with shape registration. We call this general paradigm the *alternation approach* to generalized procrustes analysis.[1] The paradigm of estimating

[1]The *alternation* term comes from the Structure-from-Motion literature where there exist methods that alternate the estimation of the camera motion with the estimation of the scene structure, see Mahamud et al. (2001) for instance.