



Efficiency assessment using slacks-based model with integer interval data

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Abstract

In this work, we present and comment some results from our recent paper [7] on efficiency on slacks-based DEA problem with integer interval data. To this matter, a new approach consisting of two phases is proposed, considering integer interval arithmetic and partial orders. The new method is illustrated and applied to a dataset from the literature.

Keywords: Efficiency, integer DEA, interval data, production possibility set, slacks-based mode

1. INTRODUCTION

Data envelopment analysis (DEA) is a non-parametric methodology for assessing the efficiency score of a set of Decision Making Units (DMU)s. DEAs are assumed to consume inputs to produce outputs and, from that, using some optimization models, one efficiency score and an efficient target can be computed for each DMU ([1], [2]). A production possibility set (PPS) is derived from some basic axioms. The non-dominated subset of the PPS is usually called the efficient frontier (EF). The efficient DMUs belong to EF while the inefficient DMUs do not belong to EF and can be projected into EF.

There are different ways to calculate the efficiency score, e.g. using efficiency potential (Lozano and Calzada-Infante [3]) or multi-directional approaches (Lozano and Soltani [4]), among others. An integer interval model was first defined by Lozano and Villa [5], and studied by other researchers, for example [6]. As an extension of these works, we consider uncertainty on the integer inputs and outputs by means of integer intervals, and comment a very recent DEA approach addressed by Arana-Jiménez et al. [7], with an axiomatic derivation of the production possibility set.

3. NOTATION AND PRELIMINARIES

Apt and Zoetewij [8] have defined the following arithmetic operations on integer intervals A and B:

1. Addition: $A+B:=\{a+b|a\in A, b\in B\}$
2. Subtraction: $A-B:=\{a-b|a\in A, b\in B\}$
3. Multiplication: $A*B:=\{a*b|a\in A, b\in B\}$
4. Multiplication by scalar: for any integer λ ,

$$\lambda*A:=\begin{cases} \lambda*A, & \&\lambda\geq 0 \\ -\lambda*A, & \&\lambda< 0 \end{cases}$$

Therefore, for A, B integer intervals and a λ an integer the following holds:

1. A+B, A-B are integer intervals.
2. A*B and $\lambda*B$ are not an integer interval.

To solve this problem, it is necessary to introduce a new multiplication operation for the multiplication between two integer interval to be an integer interval.

Let Z be the integer set. Given $\underline{a}, \bar{a} \in Z$ with $\underline{a} \leq \bar{a}$, we define the integer interval $[\underline{a}, \bar{a}]_Z$ as $\{a:a \in Z, \underline{a} \leq a \leq \bar{a}\} \subset Z$.