Conservation Laws of the Kudryashov and Sinelshchikov equation by the Scaling Method

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Abstract

In this paper, we obtain the conservation laws for Kudryashov and Sinelshchikov (K-S) equation by the scaling method. This equation appears in calculation of nonlinear pressure waves in a mixture liquid and gas bubbles. The scaling method is the algorithmic method which based on calculus, variational calculus, and linear algebra. In this method the density of conservation law is obtained by scaling symmetry and the corresponding flux is calculated by homotopy operator. We use this method to construct conservation laws of rank 6, 7 for the K-S equation

Keywords: Scaling symmetry, Conservation law, Kudryashov-Sinelshchikov (K-S) equation, Homotopy operator.

1. INTRODUCTION

Conservation laws are divergence expressions that vanish on the solutions of a partial differential equation (PDE). They appear in various scientific and research fields. Conservation laws can be considered as a mathematical description of the principles of fundamental physics and are in fact one of the most important laws in physics. They express that a certain physical quantity, e.g., energy, mass, momentum or electric charge, stays constant along time, during a physical processes. These laws also play an important role in the analysis of partial differential equations, for example in the study of the existence, uniqueness and stability of the solutions of nonlinear equations. Conservation laws are very useful in investigation of non-local systems and potential variables. There are several methods for calculating the conservation laws. An important method for calculation conservation laws uses the Noether's theorem [5, 6]. According to the Noether's method, conservation laws for a system of PDEs are derived from symmetry of the associated variational problem. Another important method for obtaining conservation laws is the scaling method, which is based on methods of differential geometry, linear algebra, variational calculus and using the homotopy formula [7, 8].
In this study, we will obtain by the scaling method, conservation laws of Kudryashov and Sinelshchiko (K-S) equation,

$$u_t + uu_x - (1+u)u_{xx} - u_x{}^2 = 0, \tag{1}$$

that is introduced by Kudryashov and Sinelshchikov in the study of nonlinear pressure waves in a mixture of a liquid and gas bubbles [4]. This paper is organized as follows. In section 2, we bring some definitions and previous studies that will be used along this work. Section 3, is devoted to Computing Conservation Laws of the K-S equation.

2. Notations and Definitions