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Volterra Integro- Differential Equations in a Hilbert Space

Leila Alizadeh
Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University,
Le_alizad@yahoo.com

Abstract:
In the Hilbert Space $\oplus_{j=1}^{n}(w_2^4[a,b]\oplus w_2^4[a,b])$. Based on the concept of the reproducing kernel function combined with Gram-schmidt orthogonalization process, we represent an exact solution in a form of fourier series in the reproducing kernel Hilbert space $\oplus_{j=1}^{n}(w_2^2[a,b]\oplus w_2^2[a,b])$. Accordingly, the approximate solution of the system of fuzzy Volterra integro-differential equations is obtained by the n-term intercept of the exact solution and proved to converge to the exact solution. Finally, two numerical examples are presented to illustrate the reliability, appropriateness and efficiency of the method.
Keywords: Hilbert space, Voltera differential equation, integral equation

1. Introduction

The fuzzy calculus represents an important introduction in applied mathematics to study numerous problems from different fields of engineering and science as including insurance, mathematical physics, finance, economics, statistics, metrology and fluid dynamics [1,2]. Due to the importance of the subject, the fuzzy system which is an important part of fuzzy calculus is a situation which involves imperfect or unknown information. It applies to forecasts of future events, to physical measurements that are as of now made, or to the unknown. One of the objectives for studying a system of fuzzy Volterra integro-differential equations is to develop the methodology of formulations and to find solotions of problems that are too complicated or ill-defined to be acceptable for analysis by conventional techniques. Many practical problems in engineering and science can be transformed into a system of fuzzy Volterra integro-differential equations; thus, their solution is one of the main goals in various areas of engineering and applied sciences.

There are two definitions of the fuzzy derivative which are the Seikkala derivative [3] differentiability to solve the system of fuzzy Volterra integro-differential equations because it gives two locally solutions while the Seikkala derivative gives one locally solution. Besides, the strongly generalized differentiability more general than the Seikkala derivative. The solution of a system of fuzzy Volterra integro-differential equations (SFVIDEs) which satisfies fuzzy initial conditions occurs in many practical problems. Generally, the approximate solutions of SFVIDEs are becoming more important because many of the linear or nonlinear SFVIDEs could not be solved exactly and sometimes it may not be possible to find their analytic solutions when the SFVIDEs are nonlinear. In [5], the numerical solution of SFVIDEs has been studied by variational iterative method (VIM) under Seikkala derivative. Recently, from the best of our knowledge, no other researchers have attempted to solve SFVIDEs under strongly generalized differentiability. The main reason of this.

2. Preliminaries in fuzzy calculus

A fuzzy number $u:\mathbb{R}\to[\mathbf{0},\mathbf{1}]$ is a generalization of a regular, real number in the sense that does not refer to one single value, but rather to a connected set of possible values, where each that value has its own weight between zero and one. This weight is called the membership function fuzzy number is thus a special case of a convex, normalized fuzzy set of the real line (see [23]).

Let R denote the family of all fuzzy numbers. For $\mathbf{0}<\alpha\le\mathbf{1}$ set $[u]_\alpha=\{\tau\in\mathbb{R}\,|\,u(\tau)\ge\alpha\}$ and $[u]_\mathbf{0}=\{\tau\in\mathbb{R}\,|\,u(\tau)>\mathbf{0}\}$ (the closure of $\{\tau\in\mathbb{R}\,|\,u(\tau)>\mathbf{0}\}$). Then the α -level set $[u]_a$ is a non-empty compact interval for all $\mathbf{0}\le\alpha\le\mathbf{1}$ and any $u\in\mathbb{R}$. In [24], if $u:\mathbb{R}\to[\mathbf{0},\mathbf{1}]$ is a fuzzy number with α -cut representation $[\underline{u}(\alpha),\overline{u}(\alpha)]$, then u satisfies the following requirements: