Astrophys Space Sci
DOI 10.1007/s10509-013-1762-1

ORIGINAL ARTICLE

Optimal strategies: theoretical approaches to the parametrization of the dark energy equation of state

Seokcheon Lee

Received: 18 November 2013 / Accepted: 18 December 2013

Abstract The absence of compelling theoretical model requires the parameterizing the dark energy to probe its properties. The parametrization of the equation of state of the dark energy is a common method. We explore the theoretical optimization of the parametrization based on the Fisher information matrix. As a suitable parametrization, it should be stable at high redshift and should produce the determinant of the Fisher matrix as large as possible. For the illustration, we propose one parametrization which can satisfy both criteria. By using the proper parametrization, we can improve the constraints on the dark energy even for the same data. We also show the weakness of the so-called principal component analysis method.

Keywords Dark energy · Parametrization · Models

We consider strategies for the most accurate determination of the dark energy (DE) parameters using cosmological observables. Due to the absence of compelling theoretical model, the parameterizing the DE by its equation of state (eos) ω is commonly used in the analysis. It is usually considered what is the optimal redshift distribution to best constrain those parameters (Tegmark et al. 1998; Huterer and Turner 2001). However, we contemplate the theoretical optimization by using the Fisher information matrix and compare the different parametrization of ω.

If one has a dataset of observable $\mathcal{O}_i$ with redshifts z_i, $i = 1, \ldots, N$, one may compute the least squares estimates of the cosmological parameters by minimizing

$$\chi^2_{\mathcal{O}} = \sum_i^N \frac{(\mathcal{O}_i - \mathcal{O}(z_i, \vec{p}))^2}{\sigma_i^2}, \tag{1}$$

where $\mathcal{O}_i$ is the measured quantity, $\mathcal{O}(z_i, \vec{p})$ is the expected value of the observable for the redshift z_i, $\vec{p}$ is the set of cosmological parameters to estimate, and σ_i is the error on the measurement. Then, the Fisher information matrix is defined to be

$$F_{lm} = \left\langle \frac{1}{2} \frac{\partial^2 \chi^2_{\mathcal{O}}}{\partial p_l \partial p_m} \right\rangle_{p_l^*, p_m^*}, \tag{2}$$

where $p^*_{l,m}$ are the parameter estimates (i.e. where $\chi^2_{\mathcal{O}}$ becomes minimum). Since the likelihood function is approximately Gaussian near the maximum likelihood (ML) point, the covariance matrix for a maximum likelihood estimator is given by

$$\left(C^{-1}\right)_{lm} = \frac{1}{2} \frac{\partial^2 \chi^2_{\mathcal{O}}}{\partial p_l \partial p_m} \bigg|_{p_l^*, p_m^*}. \tag{3}$$

The Fisher information matrix is simply the expectation value of the inverse of the covariance matrix at the ML point. From now on, we will omit the square bracket and the evaluating parameter estimates in the Fisher matrix for the convenience.

The iso-$\Delta\chi^2_{\mathcal{O}}$ contour in the parameter space is approximated by the quadratic equation

$$(\Delta\vec{p})^T F \Delta\vec{p} = \Delta\chi^2_{\mathcal{O}}, \tag{4}$$

where $\Delta\vec{p} = \vec{p} - \vec{p}^*$ is the deviation of the parameters from the fiducial value and F is the Fisher matrix given in

S. Lee (✉)
School of Physics, Korea Institute for Advanced Study, Heogiro 85, Seoul 130-722, Korea
e-mail: skylee@kias.re.kr

Published online: 04 January 2014