

# Bianchi type-III bulk viscous cosmic string model in a scalar-tensor theory of gravitation

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**Abstract** A spatially homogeneous and anisotropic Bianchi type-III space-time is considered in the presence of bulk viscous fluid containing one dimensional cosmic strings in the frame work of a scalar-tensor theory of gravity proposed by Saez and Ballester (in Phys. Lett. A 113:467, 1986). We have obtained a determinate solution of the field equations of this theory, using (i) a barotropic equation of state for the pressure and density and (ii) the bulk viscous pressure is proportional to the energy density. Some physical properties of the model are also discussed.

**Keywords** Scalar-tensor theory · Bulk viscosity · Cosmic string · Bianchi type-III model

## 1 Introduction

Scalar-tensor theories of gravitation are considered to be the most natural alternatives to Einstein's theory of gravitation. In these theories gravity is mediated by a long range scalar field in addition to the usual tensor fields present in Einstein's theory. Brans and Dicke (1961) formulated a scalar-tensor theory of gravitation which introduces an additional scalar field  $\phi$  interacting equally with all forms of matter

(with the exception of electromagnetism) besides the metric tensor  $g_{ij}$  and a dimensionless coupling constant  $\omega$ . Subsequently, Saez and Ballester (1986) proposed a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. One particularly interesting result of this theory is appearance of antigravity regime, which suggests a possible connection to the missing matter problem in non-flat FRW cosmologies. In particular Amendariz-Picon et al. (2000, 2001) related this scenario to  $k$ -essence. The gravitational field equations of Saez-Ballester (SB) theory for the combined scalar and tensor fields are (with  $8\pi G = C = 1$ )

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij} \quad (1)$$

and the scalar field  $\phi$  satisfies the equation

$$2\phi^n\phi_{;i}^{:i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \quad (2)$$

Also, we have

$$T_{;j}^{ij} = 0 \quad (3)$$

which is a consequence of the field equation (1) and (2). Here  $\omega$  and  $n$  are constants.  $T_{ij}$  is the energy tensor of the matter,  $R_{ij}$  is the Ricci tensor,  $R$  is the Ricci scalar and comma and semicolon denote partial and covariant derivatives respectively.

Scalar-tensor theories have many interesting properties and have been extensively discussed in literature. The most fruitful area of their application is in cosmology where the scalar field is often applied as quintessence field to drive accelerating phase of the universe. The analysis of the large scale cosmic microwave background fluctuations confirm

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