Astrophys Space Sci (2014) 349:501–527
DOI 10.1007/s10509-013-1622-z

ORIGINAL ARTICLE

Bianchi VI_h with variable gravitational and cosmological constants

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Received: 30 July 2013 / Accepted: 30 August 2013 / Published online: 22 September 2013

Abstract In order to study how the gravitational and the cosmological constants, G, Λ may vary, we consider two theoretical frameworks which are, a modification of the General Relativity and several scalar models (the standard, non-interacting and interacting models and their respective modifications to allow a G varying). We find exact self-similar solutions for the geometry Bianchi VI_h, (that is, the models: III, VI_0, and VI_h,). Some physical and geometrical properties of the models are also discussed and we compare the obtained theoretical results with the current observational data. In the first of the theoretical models, we reach the conclusion that, from the structure of the field equations, the behaviour of Λ and G are related, but taking into account the observational data, we conclude that the Λ behaves as a positive decreasing time function while G is growing but in the long time regimen it tends to a constant value. In the scalar models, our solutions predict a "*positive*" dynamical cosmological constant in all the obtained solutions while the behaviour of G yields indeterminate, since its depends on a free parameter, $G \approx t^{2\alpha}$, so it may be growing or decreasing as in the scalar-tensor theories.

Keywords Exact self-similar Bianchi VI · Scalar models · Time-varying constants

1 Introduction

Since the pioneers works by Dirac, who considered the possibility of a variable G (see Dirac 1938), there have been numerous modifications of general relativity to allow for variable G and Λ (see for example Barrow and Tipler 1996; Barrow and Parsons 1997). Recent observations (Perlmutter et al. 1997, 1998, 1999) suggest us that the Universe is expanding in an accelerating way and that such acceleration is governed by a positive dynamical cosmological constant. A summary of these models with time dependent cosmological "constant" is given by Overduin and Cooperstock (1998), another review of interest is given by Amendola and Tsujikawa (2010) in the framework of dark energy model. In the same way other observations have pointed out a possible variation of the gravitational constant G (Umezu et al. 2005). In particular, observations of Hulse-Taylor binary pulsar, or Type Ia supernova observations (Riess et al. 1998). For an extensive review see Uzan (2011).

In a series of recent papers (Belinchón, 2012a, 2012b, 2013a, 2013b) we have considered three theoretical models in order to study and compare how the "constants" G and Λ may vary under the self-similar hypothesis. These different theoretical frameworks are: general relativity with time varying constants (TVC), scalar cosmological models with interacting and non-interacting scalar and matter fields and TVC and different scalar-tensor theories.

We have focused our attention on the self-similar solutions since as has been pointed out by Rosquist and Jantzen (1985, 1986), self-similar models correspond to equilibrium points, playing a dominant role in the dynamics of Bianchi cosmological models (Wainwright and Ellis 1997). For this reason, Coley (2003) has stressed the fact that the self-similar models play an important role in describing the asymptotic dynamics of the more complicated Bianchi cosmological models. A large class of orthogonal spatially homogeneous models (including all class B models) are asymptotically self-similar at the initial singularity, intermediate stages of their evolution and at late times. In summary,

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