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ORIGINAL ARTICLE

Effects of zonal harmonics and a circular cluster of material points on the stability of triangular equilibrium points in the R3BP

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Abstract We have studied a modified version of the classical restricted three-body problem (CR3BP) where both primaries are considered as oblate spheroids and are surrounded by a homogeneous circular planar cluster of material points centered at the mass center of the system. In this dynamical model we have examined the effects of oblateness of both primaries up to zonal harmonic J_4; together with gravitational potential from the circular cluster of material points on the existence and linear stability of the triangular equilibrium points. It is found that, the triangular points are stable for $0 < \mu < \mu_c$ and unstable for $\mu_c \leq \mu \leq \frac{1}{2}$, where μ_c is the critical mass ratio affected by the oblateness up to J_4 of the primaries and potential from the circular cluster of material points. The coefficient J_4 has stabilizing tendency, while J_2 and the potential from the circular cluster of material points have destabilizing tendency. A practical application of this model could be the study of the motion of a dust particle near oblate bodies surrounded by a circular cluster of material points.

Keywords Restricted three-body problem · Zonal harmonic effect · Potential from a circular cluster of material points

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1 Introduction

The circular restricted three-body problem (CR3BP) specifies the dynamics of a body having infinitesimal mass and moving in the gravitational field of two bodies, called primaries, which revolve around their common centre of mass on circular orbits in accordance with the laws of motion of the two-body problem. The Earth-Moon system together with an artificial satellite constitutes such a problem. The CR3BP possesses five points of equilibrium: three collinear and two triangular, where the gravitational and centrifugal forces just balance each other. The collinear points are unstable, while the triangular points are stable for the mass ratio less than 0.03852... (Szebehely 1967).

The classical CR3BP considers the bodies to be strictly spherical, but some bodies in the solar system (e.g., Earth, Jupiter and Saturn) and in the stellar system (e.g., Regulus, Peanut binary, Antares and Altair) are sufficiently oblate. The oblateness of a body can produce perturbations- deviations from the two-body motion. This justifies the inclusion of oblateness of a body in the study of CR3BP (Vidyakin 1974; Subba Rao and Sharma 1975, Sharma and Subba Rao 1975, 1976; Sharma 1987; Kalvouridis 1993; Singh and Ishwar 1999; Kushvah 2008; Abouelmagd 2012; Singh and Taura 2013; Singh and Umar 2013).

In the stellar system there are rings of dust particles, which are regarded as the young analogues of the Kuiper belt (Greaves et al. 1998). Trilling et al. (2007) detected debris disks in many main-sequence stellar binary systems using the *Spitzer Space Telescope*. Out of an observed 69 A3-F8 main sequence binary star systems, nearly 60 % showed dust disks surrounding binary stars. It is therefore, reasonable to include the effects of additional gravitational forces from the disks on the motion of the infinitesimal mass in the CR3BP. The effect of disks is very helpful in the study

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