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ORIGINAL ARTICLE

Stability and regions of motion in the restricted three-body problem when both the primaries are finite straight segments

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Abstract In this paper we have studied the locations and stability of the Lagrangian equilibrium points in the restricted three-body problem under the assumption that both the primaries are finite straight segments. We have found that the triangular equilibrium points are conditional stable for $0 < \mu < \mu_c$, and unstable in the range $\mu_c < \mu \leq 1/2$, where μ is the mass ratio. The critical mass ratio μ_c depends on the lengths of the segments and it is observed that the range of μ_c increases when compared with the classical case. The collinear equilibrium points are unstable for all values of μ. We have also studied the regions of motion of the infinitesimal mass. It has been observed that the Jacobian constant decreases when compared with the classical restricted three-body problem for a fixed value of μ and lengths l_1 and l_2 of the segments. Beside this we have found the numerical values for the position of the collinear and triangular equilibrium points in the case of some asteroids systems: (i) 216 Kleopatra-951 Gaspara, (ii) 9 Metis-433 Eros, (iii) 22 Kalliope-243 Ida and checked the linear stability of stationary solutions of these asteroids systems.

Keywords Restricted three-body problem · Equilibrium points · Stability · Straight segments · Regions of motion · Asteroid

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1 Introduction

The restricted three-body problem is of great importance for its application in astronomy and space dynamics. Lagrange's (1772) showed that the restricted three-body problem possesses five libration points, three collinear and two triangular. Routh (1875) discussed the stability in linear sense and established that collinear libration points are unstable for any value of mass ratio μ and triangular points are stable for $0 < \mu < \mu_c = 0.03852\ldots$, and unstable in the range $\mu_c \leq \mu \leq 1/2$. The Jacobian integral of the restricted problem has many important applications. It establishes permissible regions of motion for the third body. Using this Hill (1878) has described the motion of the moon. Beside this many mathematicians and astronomers have discussed different aspects of the restricted three- body problem and made valuable contribution.

Bhatnagar (1972) considered restricted problem in a three dimensional coordinate system and studied the existence of periodic orbits of third kind. He also showed the existence of periodic orbits of collision. Sharma and Subba Rao (1975) investigated numerically the location of the libration points of the restricted three-body problem when the primaries are oblate spheroids. After this Sharma and Subba Rao (1976) have investigated numerically stationary solution of the problem when the more massive primary is an oblate spheroid.

In the recent time Broucke (2001) has studied stable orbits of planets of a binary star system in the three dimensional restricted problem. AbdulRaheem and Singh (2006) have discussed combined effects of perturbations, radiation, and oblateness on the stability of equilibrium points in the restricted three-body problem. They found that the overall effect decreases the range of stability of the triangular points. Aggarwal et al. (2006) have studied the non-linear

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