

# Vaidya black hole in non-stationary de Sitter space: Hawking's temperature

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**Abstract** In this paper we present a class of non-stationary solutions of Einstein's field equations describing embedded Vaidya-de Sitter black holes with a cosmological variable function  $\Lambda(u)$ . The Vaidya-de Sitter black hole is interpreted as the radiating Vaidya black hole is embedded into the non-stationary de Sitter space with variable  $\Lambda(u)$ . The energy-momentum tensor of the Vaidya-de Sitter black hole is expressed as the sum of the energy-momentum tensors of the Vaidya null fluid and that of the non-stationary de Sitter field, and satisfies the energy conservation law. We study the energy conditions (like weak, strong and dominant conditions) for the energy-momentum tensor. We find the violation of the strong energy condition due to the negative pressure and leading to a repulsive gravitational force of the matter field associated with  $\Lambda(u)$  in the space-time. We also find that the time-like vector field for an observer in the Vaidya-de Sitter space is expanding, accelerating, shearing and non-rotating. It is also found that the space-time geometry of non-stationary Vaidya-de Sitter solution with variable  $\Lambda(u)$  is Petrov type D in the classification of space-times. We also find the Vaidya-de Sitter black hole radiating with a thermal temperature proportional to the surface gravity and entropy also proportional to the area of the cosmological black hole horizon.

**Keywords** Vaidya solution · de Sitter space · Exact solutions · Vaidya-de Sitter solution · Energy conditions · Surface gravity

## 1 Introduction

In general relativity the Schwarzschild solution is regarded as a black hole in an asymptotically flat space. The Schwarzschild-de Sitter solution is interpreted as a black hole in an asymptotically de Sitter space with non-zero cosmological constant  $\Lambda$  (Gibbons and Hawking 1977). The Schwarzschild-de Sitter solution is also considered as an embedded black hole that the Schwarzschild solution is embedded into the de Sitter space with cosmological constant  $\Lambda$  to produce the Schwarzschild-de Sitter black hole (Cai et al. 1998). The Vaidya solution having a variable mass  $m(u)$  with retarded time  $u$  is a non-stationary generalization of Schwarzschild black hole of constant mass  $m$  (Vaidya 1999). Mallett (1985) has introduced Vaidya-de Sitter solution with constant  $\Lambda$  by making the Schwarzschild mass  $m$  variable with respect to the retarded time  $u$  as  $m(u)$  and studied the nature of the Vaidya-de Sitter space-time (Mallett 1986).

Here the idea of this paper is to propose an exact solution of Einstein's field equations describing Vaidya black hole embedded into the non-stationary de Sitter space to obtain Vaidya-de Sitter black hole with variable  $\Lambda(u)$ . This Vaidya-de Sitter solution with variable  $\Lambda(u)$  will have the limit  $m(u) = \pm(1/3)\Lambda(u)^{(-1/2)}$  of the Vaidya mass  $m(u)$  in the extreme black hole case  $9\Lambda(u)m^2(u) = 1$ , which could not explain with the constant  $\Lambda$  in Mallett (1985). This situation can be seen in the next section of this paper.

It is well known that the original de Sitter cosmological model is *conformally* flat  $C_{abcd} = 0$  space-time with *constant curvature*  $R_{abcd} = (\Lambda/3)(g_{ac}g_{bd} - g_{ad}g_{bc})$  (Hawking

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