

# Application of binary pulsars to axisymmetric bodies in the Elliptic R3BP

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**Abstract** Binary systems hosting astrophysical compact objects such as white dwarfs and/or neutron stars provide excellent test beds for studying the impact of the oblateness of the main bodies in the restricted three-body problem (R3BP). The case is investigated when the primary bodies are non-luminous, non-spherical (oblate) bodies and the third body of infinitesimal mass is also an oblate spheroid. The existence of extra solar planets orbiting these systems constitutes a three-body problem which makes them excellent models for this axisymmetric ER3BP. The positions of the equilibrium points are affected by the oblateness parameters of the three-bodies; this is shown for double neutron star binaries. The triangular points are stable for  $0 < \mu < \mu_c$ ; where  $\mu$  is the mass ratio ( $\mu \leq 1/2$ ) and  $\mu_c$  is the critical mass value influenced by the eccentricity, semi major axis and oblateness factors. The size of the region of stability increases with decreasing values of the oblateness. The oblateness of the system's bodies does not affect the nature of the stability of the collinear points since they remain unstable. Due to the almost equal masses of the primaries, our study shows that even the triangular points of these systems are unstable.

**Keywords** Celestial mechanics · Axisymmetric ER3BP · Double pulsars

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## 1 Introduction

The restricted three-body problem (R3BP) with its applications to dynamical astronomy, celestial and space mechanics is of considerable importance today and constitutes a major source of interesting theories in lunar and planetary sciences (Topputo et al. 2005; Belbruno et al. 2008; Romagnoli and Circi 2009; Bazso 2012). Let us briefly recall that the R3BP consists of two massive bodies (primaries) moving in orbits (circular or elliptic) around their common barycenter and a third body of negligible mass being influenced, but not influencing them. A typical example of the ER3BP is the motion of an asteroid under the gravitational attraction of the Sun and Jupiter. The solution to this type of problem which has been developed over the centuries from Lagrange (1772), Laplace and Delaunay (1860), Poincare (1892), Birkhoff (1927), Szebehely (1967), Danby (1988) and others, form the basis of the study of the dynamics of celestial bodies, from the computation of the ephemerides to the recent advances in flight dynamics.

A number of researches have considered the primaries and the third body to be either point masses or spherical in shape, but in general, celestial and stellar bodies are axisymmetric and they move mostly in elliptic orbits around their common center of mass. Certain planets (Earth, Jupiter and Saturn) and stars (Achernar, Alfa Arae, Regulus, VFTS 102, Vega and Altair) are sufficiently oblate for the departure from sphericity to be very significant in the R3BP. Furthermore, the fast rotation of stars (Domiciano et al. 2003; McAlister et al. 2005; Iorio 2008; Dufton et al. 2011; Van Belle et al. 2001; Meilland et al. 2009; Yoon et al. 2010) produces an equatorial bulge due to centrifugal force. As a result, neutron stars, pulsars, white and brown dwarf stars may also be oblate in shape (Arutyunyan et al. 1971; Papoyan et al. 1971; Iorio 2007a, 2007b; Hartle 1967, 1968; Laarakkers 1999; Shibata 1998; Boshkayev