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ORIGINAL ARTICLE

Noether gauge symmetry approach in quintom cosmology

Adnan Aslam · Mubasher Jamil · Davood Momeni · Ratbay Myrzakulov · Muneer Ahmad Rashid · Muhammad Raza

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Abstract In literature usual point like symmetries of the Lagrangian have been introduced to study the symmetries and the structure of the fields. This kind of Noether symmetry is a subclass of a more general family of symmetries, called Noether gauge symmetries (NGS). Motivated by this mathematical tool, in this paper, we study the generalized Noether symmetry of quintom model of dark energy, which is a two component fluid model with quintessence and phantom scalar fields. Our model is a generalization of the Noether symmetries of a single and multiple components which have been investigated in detail before. We found the general form of the quintom potential in which the whole dynamical system has a point like symmetry. We investigated different possible solutions of the system for diverse family of gauge function. Specially, we discovered two family of potentials, one corresponds to a free quintessence (phantom) and the second is in the form of quadratic interaction between two components. These two families of potential functions are proposed from the symmetry point of view, but in the quintom models they are used as phenomenological models without clear mathematical justification. From integrability point of view, we found two forms of the scale factor: one is power law and second is de-Sitter. Some cosmological implications of the solutions have been investigated.

Keywords Cosmology · Noether symmetries · Quintom fields · Dynamical systems · Cosmography · Local stability

1 Introduction

Einstein gravity inspired from the equivalence principle, follows the Mach's principle as the matter creates the geometry or space time concept. There is no reasonable and clear motive to believe that Einstein gravity must work beyond the solar system and be able to explain the large scale structure of the universe, as well as the gravity in compact objects and solar system. Just on an adhoc basis and by assuming that the equivalence principle also works on large scale, the relativistic cosmology of Einstein gravity has been constructed. Because gravity as a theory for gravitation treats like a gauge theory, cosmology based on such a gauge theory of gravity is basically a highly non-linear system of differential equations. It means even there is no uniqueness principle or theorem for the solutions of the equations. This is a weak point or very bad freedom in the theory as a mathematical point of view. Symmetry is a key point for non linear differential equations. Modern approach is how to find the general point like symmetries of a given Lagrangian (holonomic or non-holonomic). This appropriate powerful method to find and investigate the solutions of linear (non linear) dynamical systems reduces the numbers of the unknown functions by construction of invariants, the quantities which remain invariance under gauge transformations. Cosmological models are dynamical systems with attractors and integrable families. It means, if we start from an initial value of fields

A. Aslam · M. Jamil · M.A. Rashid
Center for Advanced Mathematics and Physics (CAMP), National University of Sciences and Technology (NUST), H-12, Islamabad, Pakistan

D. Momeni (✉) · R. Myrzakulov
Eurasian International Center for Theoretical Physics, Eurasian National University, Astana 010008, Kazakhstan
e-mail: momeni_d@enu.kz

M. Raza
Department of Mathematics, COMSATS Institute of Information Technology (CIIT), Sahiwal Campus, Islamabad, Pakistan