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ORIGINAL ARTICLE

# On the construction of low-energy cislunar and translunar transfers based on the libration points

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**Abstract** There exist cislunar and translunar libration points near the Moon, which are referred to as the $LL_1$ and $LL_2$ points, respectively. They can generate the different types of low-energy trajectories transferring from Earth to Moon. The time-dependent analytic model including the gravitational forces from the Sun, Earth, and Moon is employed to investigate the energy-minimal and practical transfer trajectories. However, different from the circular restricted three-body problem, the equivalent gravitational equilibria are defined according to the geometry of the instantaneous Hill boundary due to the gravitational perturbation from the Sun. The relationship between the altitudes of periapsis and eccentricities is achieved from the Poincaré mapping for all the captured lunar trajectories, which presents the statistical feature of the fuel cost and captured orbital elements rather than generating a specified Moon-captured segment. The minimum energy required by the captured trajectory on a lunar circular orbit is deduced in the spatial bi-circular model. The idea is presented that the asymptotical behaviors of invariant manifolds approaching to/traveling from the libration points or halo orbits are destroyed by the solar perturbation. In fact, the energy-minimal cislunar transfer trajectory is acquired by transiting the $LL_1$ point, while the energy-minimal translunar transfer trajectory is obtained by transiting the $LL_2$ point. Finally, the transfer opportunities for the practical trajectories that have escaped from the Earth and have been captured by the Moon are yielded by the transiting halo orbits near the $LL_1$ and $LL_2$ points, which can be used to generate the whole of the trajectories.

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## 1 Introduction

Previous research on cislunar transfer trajectories from the Earth to Moon in the context of two-body dynamics reached the conclusion that the spacecraft has to be accelerated up to the hyperbolic velocity so as to escape the Earth's gravitational force; while some recent research from the viewpoint of the restricted circular three-body problem (abbreviated CR3BP) showed that the hyperbolic velocity is not a necessary condition for the cislunar transfer (Koon et al. 2007). Compared with Hohmann transfer, the ballistically captured trajectory known as the type of low-energy transfer trajectories (Xu and Xu 2009), which is obtained within the context of CR3BP, has lower fuel consumption but longer transfer duration.

Conley studied the local dynamical behavior of planar CR3BP near the collinear libration point and classified all the trajectories into four different types as follows: periodical orbit (named Lyapunov orbit), stable/unstable manifolds of periodic orbit, transiting, and non-transiting trajectories (Conley 1968). It is concluded from Conley's work that the invariant manifolds of periodic orbits will separate transiting and non-transiting trajectories, and only the transiting ones can be employed to generate the low-energy cislunar transfer trajectories.

McGehee investigated the global dynamical behavior of CR3BP and achieved similar results, i.e., the stable and unstable manifolds of Lyapunov orbit form a two-dimensional hyper-surface in the three-dimensional Euclidean space which may play a significant role in understanding the transiting trajectories (McGehee 1969). Based