Astrophys Space Sci (2013) 348:41–56
DOI 10.1007/s10509-013-1534-y

ORIGINAL ARTICLE

Restricted three-body problem when one of the primaries is an ellipsoid

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Received: 11 February 2013 / Accepted: 15 June 2013 / Published online: 5 July 2013

Abstract This paper deals with the existence and the stability of the libration points in the restricted three-body problem when the smaller primary is an ellipsoid. We have determined the equations of motion of the infinitesimal mass which involves elliptic integrals and then we have investigated the collinear and non collinear libration points and their stability. This is observed that there exist five collinear libration points and the non collinear libration points are lying on the arc of the unit circle whose centre is the bigger primary. Further observed that the libration points either collinear or non-collinear all are unstable.

Keywords Restricted three-body problem · Libration points · Linear stability · Elliptic integrals

1 Introduction

Two bodies revolve around their center of mass in circular orbits under the influence of their mutual gravitational attraction and a third body (attracted by the previous two but not influencing their motion) moves in the plane defined by the two revolving bodies. The restricted problem of three bodies is said to describe the motion of the third body. Five libration points exist in the classical planar restricted three-body problem out of which two points are non collinear and three are collinear. The collinear libration points L_1, L_2 and L_3 are unstable for $0 \leq \mu \leq 1/2$ and the non collinear libration points L_4, L_5 are stable for the critical value of mass parameter $\mu < \mu_c = 0.03852\ldots$, Szebehly (1967). In recent times many perturbing forces such as oblateness, and radiation forces, coriolis and centrifugal forces, variation of the masses of the primaries and of the infinitesimal mass etc., have been included in the study of the restricted three body problem.

Restricted three-body problem has been studied by many scientists. Vidyakin (1974) has studied the stability in the Lyapunov sense of a particular solution, close to the Lagrange solution for the motion of three homogeneous spheroids and shown that for sufficiently small eccentricities of the meridian cross section of the interacting spheroids, and under certain conditions applied to the semimajor axes and masses of the spheroids, the solution is stable. Subbarao and Sharma (1975) discussed that if the bigger primary is an oblate spheroid whose equatorial plane coincides with the plane of motion, the triangular solutions form only nearly equilateral triangles with the primaries, and the range of the mass parameter which leads to stable triangular solutions decreases. El-Shaboury (1991) has considered a restricted problem of $2 + 2$ homogeneous axisymmetric ellipsoids such that their equatorial planes coincide with the orbital plane of the centers of mass and has shown that six of the equilibrium solutions are located about the collinear points of the restricted problem of three axisymmetric ellipsoids and studied a special case in which sixteen solutions are found in the neighborhood of the triangular Lagrangian points. Khanna and Bhatnagar (1999) have discussed the stationary solutions of the planar restricted three body problem when the smaller primary is a triaxial rigid body with one of the axes as the axis of symmetry and its equatorial plane coinciding with the plane of motion. The bigger primary is taken as an oblate spheroid and its equatorial plane is also coinciding with the plane of motion. They have shown that there exist five libration points,

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