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ORIGINAL ARTICLE

Velocity measurements in some classes of alternative gravity theories

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Abstract The general misconception regarding velocity measurements of a test particle as it approaches black hole is addressed by introducing generalized observer set. For a general static spherically symmetric metric applicable to both Einstein and alternative gravities as well as for some well known solutions in alternative gravity theories, we find that velocity of the test particle do not approach that of light at event horizon by considering ingoing observers and test particles.

Keywords Velocity in general relativity · Observer · Alternative gravity theories

1 Introduction

The radial motion of a test particle falling in a black hole is one of the key issues in general relativity. The infalling motion has been studied specifically for Schwarzschild black hole by several authors (Landau and Lifschitz 1971; Wald 1984; Bergmann 1942; Moller 1972). All of them reached the same conclusion that velocity of the infalling particle approaches that of light near the event horizon, which for the Schwarzschild case is at $r = 2M$, where M is the mass of the black hole. The observers, called static observers, are at rest with respect to the mass creating the gravitational field. They are actually the world lines on the hypersurface of orthogonal killing vector field for the metric describing the gravitational field. However there exists a common misconception that particle approaches the speed of light as it moves to the black hole horizon for all observers, but not as a limiting procedure for a static observer at r as $r \to 2M$. However if we assume that the particle approaches the event horizon at the speed of light for a static observer, as we have defined it earlier, then simple velocity composition law tells that it should approach the speed of light for all local observers as space time is locally Minkowskian.

So we have to modify our notion of velocity for a test particle near a black hole for a static observer which was done for Schwarzschild black hole (Crawford et al. 2002; Janis 1977). The notion of observer is implemented and used in various co-ordinate frames by several authors (Bolós 2006, 2011; Ellis et al. 1985).

However recently a progress has been made in obtaining trajectory around a general spherically symmetric non-rotating black hole by choosing a general metric ansatz (Chakraborty and Chakraborty 2011),

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \tag{1}$$

For this general case we find the velocity of the test particle with respect to a static observer ($r =$ constant) to be a function of $f(r)$. While for the case of a general observer such that both the observer and the test particle moves along geodesic in $\theta = \frac{\pi}{2}$ plane then the velocity of the test particle with respect to the observer to our surprise, do not depend on the choice of the function $f(r)$ provided the particle has high energy which is the most common case for astrophysical bodies, however it depends on the angular momenta which was absent in earlier works (Crawford et al. 2002). Then we have used some classes of spherically symmetric solutions in alternative gravity theories to find the relative velocity of a test particle with respect to an observer. We have discussed spherically symmetric solution in string inspired dilaton model (Garfinkle et al. 1991),

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