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ORIGINAL ARTICLE

On teleparallel version of stationary axisymmetric solutions and their energy contents

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Abstract In this study, we have investigated the geometrical and physical properties of stationary axisymmetric solutions. The expressions for the axial-vector and the gravitational energy and momentum densities are obtained in the context of teleparallel equivalent of general relativity. The obtained results are compared with that obtained previously in the context of Møller's tetrad theory of gravitation. We discussed special cases of these solutions.

Keywords Torsion axial-vector · Energy and momentum densities · Extra Hamiltonian

1 Introduction

As originally formulated by Einstein, the dynamical quantity of General Relativity (GR) is the space-time metric tensor, with components $g_{\mu\nu}$. An alternative geometrical formulation of GR has been proposed, where the tetrad $e^a{}_\mu$ is considered the basic physical variable instead of the metric tensor. This tetrad forms the orthogonal basis for the tangent space at each point of space-time and can naturally be interpreted as reference frame adapted to observer in space-time (Hehl et al. 1991).

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When working with the tetrad, one naturally encounters the notion of torsion, which can be used to describe GR entirely with respect to torsion instead of curvature derived from the metric only. This is the so-called Teleparallel Equivalent to General Relativity (TEGR) (Hayashi and Shirafuji 1979). The TEGR is a gravity theory which uses the curvature-free Weitzenböck connection (Weitzenböck 1923) to define the covariant derivative, instead of the conventional torsionless Levi-Civita connection of GR, and attempts to describe the effects of gravitation in terms of torsion instead of curvature.

It is well known that the interesting and challenging problems of GR is the energy and momentum localization and the search for a consistent expression for the gravitational energy, since no tensorial expression for the gravitational energy-momentum density can be defined in this theory. The basic reason for this impossibility is that both gravitation and inertial effects are mixed in the spin connection of the theory and cannot be separated. That is, the energy-momentum density of gravitation will necessarily include both the energy-momentum density of gravity and energy-momentum density of matter.

It has been shown that the TEGR is a suitable framework for addressing both the energy and momentum densities of the gravitation field. In the context of the TEGR we consider the stationary axisymmetric solution for addressing the distribution of gravitational energy associated to it.

For a given space-time metric tensor, there is a host of tetrad solutions that yields $g_{\mu\nu}$. The components of the Riemannian metric $g_{\mu\nu}$ of the space-time can be reformulated in terms of the tetrad $e^a{}_\mu$ in the following way

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu, \qquad (1)$$