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ORIGINAL ARTICLE

Evaporating quantum Lukewarm black holes final state from back-reaction corrections of quantum scalar fields

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Abstract We obtain renormalized stress tensor of a massless, charge-less dynamical quantum scalar field, minimally coupled with a spherically symmetric static Lukewarm black hole. In two dimensional analog the minimal coupling reduces to the conformal coupling and the stress tensor is found to be determined by the nonlocal contribution of the anomalous trace and some additional parameters in close relation to the work presented by Christensen and Fulling. Lukewarm black holes are a special class of Reissner-Nordström-de Sitter space times where its electric charge is equal to its mass. Having the obtained renormalized stress tensor we attempt to obtain a time-independent solution of the well known metric back reaction equation. Mathematical derivations predict that the final state of an evaporating quantum Lukewarm black hole reduces to a remnant stable mini black hole with moved locations of the horizons. Namely the perturbed black hole (cosmological) horizon is compressed (extended) to scales which is smaller (larger) than the corresponding classical radius of the event horizons. Hence there is not obtained an deviation on the cosmic sensor-ship hypothesis.

Keywords Hawking radiation · Lukewarm black hole · Back reaction equation · Reissner Nordström de Sitter · Noncommutative quantum gravity · Stability

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1 Introduction

Semiclassical approach of quantum gravity theory is known as quantum matter field theory propagated on a curved space-time, in which a classically treated curved space-time is perturbed by a suitable quantum matter field (Birrell and Davies 1982). A fundamental problem in this version of the quantum gravity theory, is calculation of renormalized expectation value of quantum matter stress tensor operator $\langle\hat{T}_{\mu\nu}\rangle_{ren}$. Renormalization theory give us a suitable theoretical prediction, in which expectation value of a singular quantum field stress tensor operator reduces to a nonsingular quantity contained an anomalous trace. This nonsingular stress tensor treats as source in RHS of the Einstein's gravity equation such as follows.

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi\left\{T_{\mu\nu}^{class} + \langle\hat{T}_{\mu\nu}\rangle_{ren}\right\} \tag{1}$$

where $G_{\mu\nu}$ is Einstein tensor with the perturbed metric $g_{\mu\nu} = \hat{g}_{\mu\nu} + \Delta g_{\mu\nu}$ and the background metric $\hat{g}_{\mu\nu}$, Λ is positive cosmological constant and $T_{\mu\nu}^{class}$ is classical baryonic matter or non-baryonic dark matter field stress tensor. Non-minimally coupled scalar dark matter fields with a negative value of equation of state parameter may to be come originally from effects of conformal frames. The latter case of the matter is a good candidate to explain positivity accelerated expansion of the universe and to remove the naked singularity of the universe in quantum cosmological approach. See Nozari and Sadatian (2009) and references therein. The above equation which is written in units $G = \hbar = c = 1$ is called the metric back-reaction equation. There are presented several methods for the renormalization prescription, namely dimensional regularization, point splitting, adiabatic and Hadamared renormalization prescriptions (Birrell and Davies 1982). The latter method