Astrophys Space Sci (2013) 346:443–461
DOI 10.1007/s10509-013-1473-7

ORIGINAL ARTICLE

Stability of the classical type of relative equilibria of a rigid body in the J_2 problem

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Received: 20 March 2013 / Accepted: 24 April 2013 / Published online: 31 May 2013

Abstract The motion of a point mass in the J_2 problem is generalized to that of a rigid body in a J_2 gravity field. The linear and nonlinear stability of the classical type of relative equilibria of the rigid body, which have been obtained in our previous paper, are studied in the framework of geometric mechanics with the second-order gravitational potential. Non-canonical Hamiltonian structure of the problem, i.e., Poisson tensor, Casimir functions and equations of motion, are obtained through a Poisson reduction process by means of the symmetry of the problem. The linear system matrix at the relative equilibria is given through the multiplication of the Poisson tensor and Hessian matrix of the variational Lagrangian. Based on the characteristic equation of the linear system matrix, the conditions of linear stability of the relative equilibria are obtained. The conditions of nonlinear stability of the relative equilibria are derived with the energy-Casimir method through the projected Hessian matrix of the variational Lagrangian. With the stability conditions obtained, both the linear and nonlinear stability of the relative equilibria are investigated in details in a wide range of the parameters of the gravity field and the rigid body. We find that both the zonal harmonic J_2 and the characteristic dimension of the rigid body have significant effects on the linear and nonlinear stability. Similar to the classical attitude stability in a central gravity field, the linear stability region is also consisted of two regions that are analogues of the Lagrange region and the DeBra-Delp region respectively. The nonlinear stability region is the subset of the linear stability region in the first quadrant that is the analogue of the Lagrange region. Our results are very useful for the studies on the motion of natural satellites in our solar system.

Keywords J_2 problem · Rigid body · Non-canonical Hamiltonian structure · Relative equilibria · Linear stability · Nonlinear stability

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1 Introduction

The J_2 problem, also called main problem of artificial satellite theory, in which the motion of a point mass in a gravity field truncated on the zonal harmonic J_2 is studied, is an important problem in the celestial mechanics and astrodynamics (Broucke 1994). The J_2 problem has its wide applications in the orbital dynamics and orbital design of spacecraft. This classical problem has been studied by many authors, such as Broucke (1994) and the literatures cited therein.

However, neither natural nor artificial celestial bodies are point masses or have spherical mass distributions. One of the generalizations of the point mass model is the rigid body model. Because of the non-spherical mass distribution, the orbital and rotational motions of the rigid body are coupled through the gravity field. The orbit-rotation coupling may cause qualitative effects on the motion, which are more significant when the ratio of the dimension of rigid body to the orbit radius is larger.

The orbit-rotation coupling and its qualitative effects have been discussed in several works on the motion of a rigid body or gyrostat in a central gravity field (Wang et al. 1991, 1992, 1995; Teixidó Román 2010). In Wang and Xu